Dimensional Analysis of the Effect of Wind Speed on Corona Discharge Current

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Abstract: *Corona discharge is a phenomenon arising in high voltage systems, which involves many physical variables, and is not amenable to satisfactory analysis via conventional methods. One accessible and simple way that is proven to determine (at least qualitatively) how the corona discharge current is related to the physical quantities that influence it is that of Dimensional Analysis. This paper reviews the use of Dimensional Analysis in the derivation of dimensionless products related to corona discharges for a few different applications. However, the main contribution of the paper is to use Dimensional Analysis to investigate the nature of the corona discharge current subject to the effect of wind speed. This latter analysis might be thought of as an umbrella for the previous analyses as it joins or encompasses the influencing variables used in the former cases. Computational details of this latter analysis is exposed in a tutorial fashion. By using qualitative analysis based on dimensional grounds, we show that the corona discharge current dependence on wind speed is very complex, indeed. However, under certain simplifying assumptions, we demonstrate that the corona discharge current is directly proportional to wind speed.*

Keywords: *Corona discharges, dimensional analysis, wind speed*

1. Introduction

Corona discharges are transient or steady-state discharges due to the non-uniform electric field resulting from the presence of high electric voltages associated with the existence of suitable metallic structures[1]. This phenomenon occurs in gases at high pressures and temperatures, in particular in helium, nitrogen, synthesis gas (gas mixture consisting primarily of hydrogen and carbon monoxide), and other gases under high voltage. The presence of this phenomenon might be detected via certain visible and audible cues that can be used as indicators to evaluate the performance of some related applications. The research concerning the physical mechanism of corona discharge is still in progress by many researchers and investigators worldwide. The current challenge is how to derive a general physical interpretation of the diverse phenomena associated with corona discharges.

There are many applications associated with corona discharge phenomena. Some of these are advantageous and the others are disadvantageous. Electronic precipitators constitute the most important application that uses the corona discharge phenomenon to advantage. Moreover, the visual cue of corona discharges is usually used to analyze the effectiveness of a lightning rod tip. On the other hand, the negative effect of corona discharges is influential in the detrimental ozone generation due to the corona discharges in electronic devices such as laser printers, photocopiers and electrostatic precipitators which is harmful to the human respiratory system and to lung function.

Dimensional Analysis is used herein as a tool to analyze the physical interpretation of corona discharges, thereby opening the door for investigating the role played by this phenomenon in many applications. Generally, Dimensional Analysis is an effective way to analyze a physical system without explicit knowledge of its governing physical laws[2]. Dimensional analysis can be used to determine the relationship among physical quantities using only their physical dimensions and can provide the general forms of equations that describe natural phenomena. By using this method of analysis, we can reduce a complex physical problem to a simple form that can surrender to quantitative analysis by subsequent experimental investigation.

Dimensional Analysis is limited by its incapability to decide whether there are any missing variables among the physical quantities included in a functional solution. So, it is important to involve as many variables as possible in the dimensional analysis process. The relation among the many variables involved is derived by alluring to the existence of m dimensionless products (Φ) comprised of the pertinent variables raised to certain positive, zero or negative exponents. This existence of dimensionless products follows from the Buckingham theorem[3], which states that if there exists a dimensionally homogenous equation

$$F(A_1, A_2, \ldots, A_n) = 0 \quad (1)$$

among n physical quantities, which involves k physical dimensions (k < n), then there also exists a relation

$$\Phi(\pi_1, \pi_2, \ldots, \pi_m) = 0, \quad (2)$$

among m independent dimensionless products π_i comprised each of the pertinent variables A_j raised to certain positive, zero, or negative exponents, where

$$(n-k) \leq m < n \quad (3)$$

In fact, m is equal to $(n-r)$, where r $(0 < r \leq k)$ is the *rank* of the k by n *dimensional matrix* formed by the dimensional exponents of the A_i's . This matrix has rows indexed by the physical dimensions, columns indexed by the variables, and its typical element is the exponent to which the pertinent physical dimension is raised in the pertinent variable [2]. Another key theory is the *Product Theorem* [2], which states that: "If a secondary quantity is derived from measurements of primary quantities *X, Y, Z, . . .* ; assuming absolute significance of relative magnitudes, the value of the secondary quantity is derived as: $C X^a Y^b Z^c \ldots$, where *C, a, b, c, . . .* are constants."

This paper discusses the dimensional analysis of different phenomena of corona discharges treated in various sources in the open literature. A notable contribution of this paper is to build a larger physical system that involves more variables in the case of the effect of wind speed on the corona discharge current in a lightning rod.

The rest of the paper has the following organization. Section II presents concise results (omitting detailed computations) for the dimensional analysis of various applications on corona discharges that are already available in the open literature. Section III presents a novel and large problem of dimensional analysis that acts as an umbrella for the previous analyses as it joins or encompasses the variables used in the former cases. Detailed computationally-efficient techniques are given explicitly in a tutorial expository fashion. Section IV discusses the qualitative analysis for the main result on section III. Section V concludes the paper.

2. Literature Review

2.1 Lightning rod tips (first treatment)

Performance of various shapes of lightning rod tips can be evaluated based on their corona discharge current and breakdown characteristics. Corona discharge current in lightning rod tips is the leakage current that is created due to the excess space charges surrounding rod tips that will be attracted to the grounding. Corona discharge can be used to clarify the effect of the tip radius and gap length in the inception regime for the simple electrode geometry of a spherically tipped cylindrical rod[4].

The current literature has very few papers that analyze the effectiveness of rod tips based on corona discharge using dimensional analysis. Specifically, the authors in [5] observe the corona discharge pattern on four different lightning rods tip structures. To analyze further, the authors developed a mathematical model based on their observations so as to handle their problem via Dimensional Analysis. Six physical variables involved in the mathematical model developed are shown in Table 1. For convenience, we use the dimensional basis employed in [6], which is the MLTQ basis utilizing the dimensions of mass, length, time, and electric charge. Alternatively, we could have used the MLTI basis, which replaces the dimension of electric charge by that of the electric current. Yet a more convenient basis is the LTIϕ basis, which further replaces the dimension of mass by that of the electric potential or voltage.

Table 1 Symbols, units and dimensions of relevant variables in the problem of lightning rod tips

Variable	Symbol	Unit	Dimension
Corona discharge current	I_{cd}	Ampere	$M^0 L^0 T^{-1} Q^1$
Permittivity	ϵ	F/m	$M^{-1} L^{-3} T^2 Q^2$
Time	T	Second	$M^0 L^0 T^1 Q^0$
Area of the tip that produces corona	A	Meter^2	$M^0 L^2 T^0 Q^0$
Distance	d	Meter	$M^0 L^1 T^0 Q^0$
Input voltage	V	Volt	$M^1 L^2 T^{-2} Q^{-1}$

All six variables can be represented in the dimensional matrix below twice: (a) for the initial situation in which each column replicates the exponents for its variable in Table 1, and (b) for the same matrix after implementation of the Gauss-Jordan algorithm, which results in the rows inheriting the dimensions of the basis variables (here, the four variables, I_{cd}, ϵ, t, and A). With this viewpoint, we might think of the Gauss-Jordan procedure as a convenient means of switching from the original familiar QTLM dimensional basis to a final $I_{cd}\epsilon tA$ dimensional basis (that is almost unheard of).

	I_{cd}	ϵ	T	A	d	V
Q	1	2	0	0	0	-1
T	-1	2	1	0	0	-2
L	0	-3	0	2	1	2
M	0	-1	0	0	0	1
I_{cd}	1	0	0	0	0	1
ε	0	1	0	0	0	-1
t	0	0	1	0	0	1
A	0	0	0	1	0.5	- 0.5

It is worth noting that the dimension of a variable is dictated by the column underneath it with respect to the current dimensional basis. For example, the variable V is characterized by the column of exponents $[-1\ -2\ 2\ 1]^T$ in the QTLM basis, and by $[1\ -1\ 1\ -0.5]^T$ in the $I_{cd}\epsilon tA$ basis. Therefore, we might express its dimension [V] as:

$[V] = [Q]^{-1}[T]^{-2}\ [L]^2\ [M]^1 = [I_{cd}]^1\ [\epsilon]^{-1}\ [t]^1\ [A]^{-0.5}$(4)

In the final dimensional matrix shown above, there are two regime variables (variables other than the basis variables), namely d and V. This matrix can be rearranged (by writing the negative transpose of the last two columns in the final dimensional matrix, augmented by a two by two unit matrix) so as to produce two dimensionless products, expressed as shown in equations (5a) and (5b):

	I_{cd}	ϵ	t	A	d	V
π_1	0	0	0	-0.5	1	0
π_2	-1	1	-1	0.5	0	1

$$\pi_1 = A^{-0.5}d \tag{5a}$$

$$\pi_2 = I_{cd}^{-1}\epsilon t^{-1}A^{0.5}\ V \tag{5b}$$

Note that Equations (5) are simply a restatement of the dimension of regime variables in terms of those of basis variable. For example, Eq. (4b) is a restatement of [V] via

$$1 = [\pi_2] = [I_{cd}]^{-1}\ [\epsilon]^1\ [t]^{-1}\ [A]^{0.5}\ [V] \tag{6}$$

According to the Buckingham theorem, the dimensionless parameters are inter-related as shown in equation (7) :

$$\Phi\ (\pi_1\ ,\pi_2) = 0, \tag{7}$$

Finally the mathematical model of the corona discharge current can be found from equation (8):

$$\pi_2 = \Psi\ (\pi_1) \tag{8a}$$

where Ψ is an unknown function. The solution of I_{cd} follows in terms of it as

$$I_{cd}^{-1}\epsilon t^{-1}A^{0.5}\ V = \Psi(A^{-.05}d)$$

$$I_{cd} = \frac{\epsilon A^{.05}\ V}{t}\ /\ \Psi(A^{-.05}d) \tag{8b}$$

2.2 Lightning rod tips (second treatment)

Another research work related to corona discharge in light rod tips is presented in [4][7]. In this paper, the authors clarified the effect of the tips radius and gap length on the interception regime for the simple electrode geometry of the lightning rod tip. Five variables involved in this model are shown in Table 2.

Table 2 Symbols, units and dimensions of relevant variables in the problem of lightning rod tips taken from [4]

Variable	Symbol	Unit	Dimension
Input Voltage	V	Volt	$M^1\ L^2\ T^{-2}Q^1$
Corona discharge	I	ampere	$M^0\ L^0\ T^{-1}Q^1$
Permittivity	ϵ	F/meter	$M^{-1}\ L^{-3}\ T^2\ Q^2$
Average mobility value	μ	Meter/second	$M^{-1}\ L^0\ T^1\ Q^1$
Separation of electrode	d	Meter	$M^0\ L^0\ T^1\ Q^0$

All five variables can be represented in the dimensional matrix below twice: (a) for the initial situation in which each column replicates the exponents for its variable in Table 2, and (b) for the same matrix after implementation of the Gauss-Jordan algorithm, which results in the rows inheriting the dimensions of the basis variables (here, the four variables, V,I,ϵ, μ, and d).

	V	I	ϵ	μ	D
M	1	0	-1	-1	0
L	2	0	-3	0	1
T	-2	-1	2	1	0
Q	-1	1	2	1	0
V	1	0	0	0	2
I	0	1	0	0	-1

ϵ	0	0	1	0	1
μ	0	0	0	1	1

Once more, we note that the dimension of a variable is dictated by the column underneath it with respect to the current dimensional basis. For example, the variable d is characterized by the column of exponents $[0\ 1\ 0\ 0]^T$ in the MLTQ basis, and by $[2\ -1\ 1\ 1]^T$ in the VIϵμ basis. Therefore, we might express its dimension [d] as:

$$[d] = [M]^0\ [L]^1\ [T]^0\ [Q]^0 = [V]^2\ [I]^{-1}\ [\epsilon]^1\ [\mu]^1 \tag{9}$$

In the final dimensional matrix shown above, there is a single regime variable, namely d. This matrix can be rearranged (by writing the negative transpose of the last column in the final dimensional matrix, augmented by a one by one unit matrix) so as to produce a single dimensionless product, expressed as shown below:

	V	I	ϵ	μ	D
π_1	-2	1	-1	-1	1

This dimensionless product π_1 is given explicitly by equation (10)

$$\pi_1 = \frac{I\ d}{\mu\ \varepsilon\ V^2} \tag{10}$$

Note that Equation (10) is simply a restatement of the dimension of the regime variable in terms of those of the basis variable. In fact, (10) is a restatement of [d] via

$$1 = \pi_1 = [V]^{-2}\ [I]^1\ [\epsilon]^{-1}\ [\mu]^{-1}\ [d]^1 \tag{11}$$

According to the Buckingham theorem, this only dimensionless parameter satisfies an equation of the form

$$\Phi\ (\pi_1) = 0 \tag{12}$$

which means that π_1 must be equal to a certain constant, say k. Finally the mathematical model of the corona discharge current is given by the formula in forthcoming equation (13):

$$I = k\,\frac{\epsilon\ \mu\ V^2}{d} \tag{13}$$

This case with only a single dimensionless product (with a necessarily constant value) is one in which DA is dramatically effective. Note that our expression for a desired output variable is complete (apart from the need to determine a single arbitrary constant). In particular, the final equation does not involve any unknown function. Another advantage gained with a single dimensionless product is that any variable (not necessarily a regime one) automatically serves as an output without further mathematical manipulations[8]. If a phenomenon could have been described by a single dimensionless product, then this product would be elevated to the distinguished status of a named constant, as it would be given the name of its discoverer.

2.3 Ozone Generation (third treatment)

Use of DC current corona discharge has led to environmental concern on the emission of detrimental ozone, which is notorious for its grave effects on vegetation, materials, and human health[9]–[12]. In fact, corona discharge of indoor electrostatic devices such as laser printers, photocopiers and electrostatic precipitators is responsible for the generation of significant amounts of detrimental ozone, which is harmful, in particular, to the human respiratory system and to the human lung function [13], [14]. The prediction of ozone formation is important to the appropriate practical design of the afore-mentioned electric devices. The present technique of dimensional analysis can be used to more accurately predict the ozone production rate as a function of the discharge and other parameters such as electrode size, temperature, air velocity and relative humidity.

There are two ways to prove a correlation between the ozone generation rate and a series of design or operating parameters for the indoor electrostatic devices. The first way is by using positive wire-to-plate corona discharge [15], while the second way is by using negative wire-to-plate corona discharge [16]. The Authors in [15], [16] developed a theoretical equation to predict the ozone generation of wire-to-plate positive corona discharge in the air by using dimensional analysis. There are seven variables that are involved in this model, namely r, a, V, d, Ve, ϵ, and μ. The names, symbols, units and dimensions (with respect to the MLTA basis) are shown in Table 3.

Table 3 Symbols, units and dimensions of relevant variables of the detrimental ozone phenomenon

Variable	Symbol	Unit	Dimension
Inter electrode gap	d	mm	$M^0\ L^1\ T^0\ A^0$
Excess voltage	Ve	Volt	$M^1\ L^2\ T^{-3}\ A^{-1}$
Permittivity	ϵ	Fm^{-1}	$M^{-1}\ L^{-3}\ T^4\ A^2$
Ion Mobility	M	M/s^2	$M^{-1}\ L^0\ T^2\ A^0$
Ozone generation rate per unit length of wire	R	$mg(sm)^{-1}$	$M^1\ L^{-1}\ T^{-1} A^0$
Wire radius	A	mm	$M^0\ L^1\ T^0\ A^0$
Applied voltage	V	volt	$M^1\ L^2\ T^{-3}\ A^{-1}$

All seven variables can be represented in the dimensional matrix below twice: (a) for the initial situation in which each column replicates the exponents for its variable in Table 3, and (b) for the same matrix after implementation of the Gauss-Jordan algorithm, which results in the rows inheriting the dimensions of the basis variables

	d	Ve	ϵ	μ	r	a	V
M	0	1	-1	-1	1	0	1
L	1	2	-3	0	-1	1	2
T	0	-3	4	2	-1	0	-3
A	0	-1	2	1	0	0	-1

D	1	0	0	0	0	1	0
Ve	0	1	0	0	1	0	1
ϵ	0	0	1	0	1	0	0
μ	0	0	0	1	-1	0	0

Again, the dimension of any variable is dictated by the column underneath it with respect to the current dimensional basis. For example, the variable V is characterized by the column of exponents $[1\ 2\ -3\ -1]^T$ in the MLTA basis, and by $[0\ 1\ 0\ 0]^T$ in the $dV_e\ \epsilon\mu$ basis. Therefore, we might express its dimension [V] as:

$[V] = [M]^{-1}\ [L]^{-2}\ [T]^{-3}\ [A]^{-1} = [d]^0\ [Ve]^1\ [\epsilon]^0\ [\mu]^0$

In the final dimensional matrix shown above, there are three regime variables (variables other than the basis variables), namely r, a and V. This matrix can be rearranged (by writing the negative transpose of the last three columns in the final dimensional matrix, augmented by a three by three unit matrix) so as to produce three dimensionless products, expressed as shown in below in matrix form, and more explicitly as equations (14a) - (14c):

	d	Ve	ϵ	μ	r	a	V
π_1	0	-1	-1	1	1	0	0
π_2	-1	0	0	0	0	1	0
π_3	0	-1	0	0	0	0	1

$$\pi_1 = \frac{r\ \mu}{\epsilon V_e} \tag{14a}$$

$$\pi_2 = \frac{a}{d} \tag{14b}$$

$$\pi_3 = \frac{V}{V_e} \tag{14c}$$

Note that Equations (14) are simply a restatement of the dimension of regime variables in terms of those of basis variable. For example, (14c) is a restatement of [V] via

$$1 = [\pi_3] = [Ve]^{-1}[V]^1 \tag{15}$$

According to Buckingham theorem, the three dimensionless parameters are related by the forthcoming equation (16):

$$\Phi\ (\pi_1\ , \pi_2\ , \pi_3) = 0, \tag{16}$$

Finally the mathematical model of ozone generation can be given by the equation (17) :

$$\pi_1 = \Psi\ (\pi_2, \pi_3) \tag{17}$$

whereΨ is an unknown function, in terms of which we can finally express the ozone generation rate r as

$$\frac{r\ \mu}{\epsilon V_e} = \Psi\left(\left(\frac{a}{d}\right)\left(\frac{V}{V_e}\right)\right) \tag{18a}$$

$$r = \left(\frac{\epsilon V_e}{\mu}\right) / \Psi(\left(\frac{a}{d}\right), \left(\frac{V}{V_e}\right)) \tag{18b}$$

2.4 Electronic precipitators (fourth treatment)

The basic principle underlying the removal process of solid pollutants is to charge the particular matter by means of corona-generated ions, which then move toward the collecting plates under the effect of the applied electric field. Systems that use this phenomenon as a way to remove solid pollutants are called electrostatic precipitators (ESP) [17]–[19]. In practice, the ESP configuration for generating the electric field involves a high voltage applied to a row of emitting wires centrally placed between two parallel earthed collecting plates. The cross-section of the wires may be round, square or further complicated in order to increase the efficiency of generating the local electrical field and the resulting corona discharge. The corona discharge in the case of negative high voltage applied to the discharge electrodes is known to be stationary.

Gutiérrez Ortiz et al. [20], [21] used seven variables to analyze the electrostatic precipitators based on dimensional analysis. These variables are listed in Table 4

Table 4 Symbols, units and dimensions of relevant variables for the electronic precipitators problem

Variable	Symbol	Unit	Dimension
Particle concentration at ESP outlet (variable related to unit efficiency, i.e., the target)	C_0	kg/m³	$M^1\ L^{-3}\ T^0 I^0$
Particle resistivity	ρ_v	Ω	$M^1\ L^2\ T^{-3}\ I^{-2}$
Migration speed of	W	m/s	$M^0\ L^1\ T^{-1}\ I^0$

particles			
Turbulent mixing coefficient (eddy diffusivity)	D_t	m^2/s	$M^0 L^2 T^{-1} I^0$
Particles density	ρ_p	kg/m^3	$M^1 L^{-3} T^0 I^0$
Density of the deposited dust layer	ρ_{dl}	kg/m^3	$M^1 L^{-3} T^0 I^0$
Particle mean size	dp_{av}	m	$M^0 L^1 T^0 I^0$

All seven variables can be represented in the dimensional matrix below twice: (a) for the initial situation in which each column replicates the exponents for its variable in Table 4, and (b) for the same matrix after implementation of the Gauss-Jordan algorithm, which results in the rows inheriting the dimensions of the basis variables (here, the four variables, C_0,ρ_v, W, and D_t).

	C_0	ρ_v	W	D_t	ρ_p	ρ_{dl}	dp_{av}
M	1	1	0	0	1	1	0
L	-3	2	1	2	-3	-3	1
T	0	-3	-1	-1	0	0	0
I	0	-2	0	0	0	0	0

C_0	1	0	0	0	1	1	0
ρ_v	0	1	0	0	0	0	0
W	0	0	1	0	0	0	1
D_t	0	0	0	1	0	0	-1

Once more, we stress that the dimension of a variable is expressed by the column underneath it with respect to the current dimensional basis. For example, the variable dp_{av} is characterized by the column of exponents $[0\ 1\ 0\ 0]^T$ in the MLTI basis, and by $[0\ 0\ 1\ -1]^T$ in the $C_0\ \rho_v W D_t$ basis. Therefore, we might express its dimension [V] as:

$$[dp_{av}] = [M]^0\ [L]^1\ [T]^0\ [I]^0 = [C_0]^0\ [\rho_v]^0 [W]^1\ [D_t]^{-1} \quad (19)$$

In the final dimensional matrix shown above, there are three regime variables (variables other than the basis variables), namely ρ_p, ρ_{dl} and dp_{av}. This matrix can be rearranged to produce two dimensionless products, expressed as shown in equations (20a) - (20c):

	C_0	ρ_v	W	D_t	ρ_p	ρ_{dl}	dp_{av}
π1	-1	0	0	0	1	0	0
π2	-1	0	0	0	0	1	0
π3	0	0	-1	1	0	0	1

$$\pi_1 = \frac{\rho_p}{C_0} \quad (20a)$$

$$\pi_2 = \frac{\rho_{dl}}{C_0} \quad (20b)$$

$$\pi_3 = \frac{D_t\ dP_{av}}{w} \quad (20c)$$

Note that Equations (20) are simply a restatement of the dimension of regime variables in terms of those of basis variable. For example, (20c) is a restatement of $[dp_{av}]$ in (19) via

$$1 = [\pi_3] = [W]^{-1}\ [D_t]^1 [dP_{av}]^1 \quad (21)$$

According to the Buckingham theorem, the dimensionless parameter are related in equation (22) :

$$\Phi\ (\pi_1, \pi_2, \pi_3) = 0 (22)$$

Finally. the mathematical model is given by the formula in equation (23):

$$\pi_3 = \Psi\ (\pi_1, \pi_2) \quad (23)$$

whereΨ is an unknown function, in terms of which we can finally express the migration speed of particles via

$$\frac{D_t dP_{av}}{w} = \Psi\ (\frac{\rho_p}{C_0}, \frac{\rho_{dl}}{C_0}) \quad (24a)$$

$$w = \frac{1}{D_t dP_{av}} / \Psi (\frac{C_0}{\rho_p}, \frac{C_0}{\rho_{dl}}) \quad (24b)$$

3. Effect of Wind of the Corona Discharges in a Rod Lightning Tip

Present the corona discharge current is influenced by the ion velocity, whose magnitude and direction both depend on the ambient air condition, including the strength and direction of wind in the surrounding atmosphere. There are some researches that assess wind affect for the electricity phenomena, T. In fact, wind affects the corona discharge by blowing away ions with airflow. It was shown analytically and numerically in [22] that even a moderate wind velocity leads to hundreds per cent increase in the corona discharge current. It is also found in [7] that the corona discharge current increases almost linearly with the increase of the wind speed and decreases rapidly when the background electric field reaches a constant value in the absence of wind.

The effect of wind also influences the efficiency of a protective lightning rod, since any effect on the corona discharge due to the effect of wind could change the efficiency of the lightning rod to protect against lightning and also the number of lightning strikes it can ultimately withstand. Aleksandrovsk et, al [23] explored the current characteristics of corona discharges related to the charge of a single charged ion (e), the air density (ρ) and the permittivity (ε) of the surrounding medium. Bazeliyan et al. [22] point out that the quantitative effect of wind is controlled by the ratio between the magnitude of wind velocity (w) and that of the ion drift velocity (μ) in the local electric field (E) produced by the corona space charge and thundercloud charges and by the directions of these velocities. Bazeliyan et al. [22] also estimated the effect of the wind speed on a non-stationary glow corona discharge. In [24], Nguyen et al, proposed a methodology to solve numerically for the drift region of corona discharge. In [25], D'Alessandro analyzed the effect of greatly reduced air density (ρ), humidity (h) and wind speed to the corona discharge current, while Belhadi et al. [26] obtained experimental data for the corona discharge related to wind velocity. Liu et al. [27] reported the dependence of the corona onset on the pertinent pressure and the ambient humidity. Guo et al [28] found that under the same wind speed, a larger horizontal wind can result in less accumulation of corona discharge, a larger electric field and a larger corona current, while in the same wind direction, a larger wind speed can cause a larger corona effect.

As far as we know, there is no paper in the open literature that used dimensional analysis to derive the physical model of the effect of wind for the corona discharge current. Most of the authors analyze the corona discharge only partially. By contrast, in the present section, we employ dimensional analysis to derive a physical model of corona discharges under the effect of wind. Our present analysis involves all the variables that influence or participate in the production of the current of corona discharges.

In this section, three methods will be used to utilize 11 relevant quantities listed in Table 5. These three methods will be discussed in subsections III.1, III.2 and III3, respectively. One of the 11 quantities considered is a general quantity of interest (Current) while the other ten are the quantities that influence the quantity of interest. The dimension of each of the eleven quantities is expressed in terms of the fundamental dimensions of mass (M), length (L) time (T) and current (I). The derivation is geared towards expressing the corona discharge current (as a dependent variable) in terms of the other ten quantities (as potentially determining or influencing variables). Furthermore, by using the resulting expression, the mathematical link between the corona discharge current and the wind velocity can be determined.

Table 5 Symbols, units and dimensions of eleven relevant variables for the wind effect on corona discharges

No	Variable	Symbol	Unit	Dimension
1	Corona discharge current	I_{cd}	Ampere	$M^0 L^0 T^0 I^1$
2	Air Permittivity	ε	F/m	$M^{-1} L^{-3} T^4 I^2$
3	Time	t	second	$M^0 L^0 T^1 I^0$
4	Area of the tip that produce corona	A	meter^2	$M^0 L^2 T^0 I^0$
5	Distance separation of electrode	d	meter	$M^0 L^1 T^0 I^0$
6	Input voltage	V	volt	$M^1 L^2 T^{-3} I^{-1}$
7	Ion drift velocity	μ	m/s	$M^0 L^1 T^{-1} I^0$
8	Wind Velocity	w	m/s	$M^0 L^1 T^{-1} I^0$
9	Air Humidity	h	kg/m^3	$M^1 L^{-3} T I^0$
10	Electric field shielded intensity	E	volt/m	$M^1 L^1 T^{-3} I^{-1}$
11	Air density	ρ	kg/m^3	$M^1 L^{-3} T^0 I^0$

In this section, the solution of the dimensional matrix is obtained by three methods that are presented in the three subsections III.1, III.2, and III3, respectively. Subsection III1 presents dimensional analysis via Gauss-Jordan Elimination, and subsection III.2 uses a direct matrix method, while subsection III.3 uses a separation matrix method, which might be depicted also as one of fundamental modes.

3.1. Using Gaussian Elimination Method

Based upon Buckingham theorem, each dimensionless product of the set of eleven quantities in Table 5 will be of the form

$$\pi = k\, w^a\, V^b\, A^c\, \varepsilon^d\, E^e\, d^f \mu^g\, \rho^h\, h^i t^j\, I_{cd}^k \qquad (25)$$

where k is a dimensionless constant, while a, b, c, d, e, f, g, h, i, j, and k are exponents yet to be partially determined or inter-related. We are particularly interested in assessing the dependence of the corona discharge current I_{cd} on wind speed (w). Therefore, we guaranteed that the former variable is a regime variable by placing it as the last variable in (25), and ensured that the latter variable is a basis variable by situating it as the first variable in (25). We note that [π] = [k] = 1,

where:

$$[\pi] = (L^1T^{-1})^a (M^1L^2T^{-3}I^{-1})^b (L^2)^c\, (M^{-1}L^{-3}T^4I^2)^{d^k} (M^1L^1T^{-3}I^{-1})^e (L)^f (L^1T^{-1})^g (M^1L^{-3})^h (M^1L^{-3})^i (T)^j \qquad (26)$$

Or equivalently :

$$L^0T^0M^0I^0 = L^{a+2b+2c-3d+e+f+g-3h-3i}\, T^{-a-3b+4d-3e-g+j}\, M^{b-d+e+h+i} I^{-b+2d-e+k} \qquad (27)$$

The product π is dimensionless if :

$$a + 2b + 2c - 3d + e + f - 3h - 3i = 0 \qquad (28a)$$

$$-a - 3b + 4d - 3e - g + j = 0 \qquad (28b)$$

$$b - d + e + h + i = 0 \qquad (28c)$$

$$-b + 2d - e + k \qquad (28d)$$

In Equations (28) we have eleven unknowns (exponents a, b, c, d, e, f, g, h, i, j, k), but only four equations that will be see shortly to be linearly independent. The dimensional matrix derived from these equations has a full rank of four, thus we can determine only four of the unknowns in term of the remaining seven. There are $\binom{11}{4}$ ways for choosing four variables out of eleven (without order or replacement). The four variables are used as basis (input) variables, and the seven variables are use as isolated (regime or output) variables. This section explores the case of expressing w ,V, A, and ε as basis variables and E, d μ, ρ, h, t, and I_{cd} as isolated/regime variables.

All variables can be represented in the dimensional matrix in Table 6 below, where details of Gauss-Jordan elimination are explicitly shown. To facilitate reading the table, we use the left column of the table to depict the elementary row operation performed at every step. Since we need to express Icd in terms of other variables, we initially placed its column at the end.

Table 6. Stages of the procedure of Gauss-Jordan elimination for the problem in Sec. III

	a	b	c	d	e	f	g	h	i	j	k	
$E1^{(1)}$	1	2	2	-3	1	1	1	-3	-3	0	0	0
$E2^{(1)}$	-1	-3	0	4	-3	0	-1	0	0	1	0	0
$E3^{(1)}$	0	1	0	-1	1	0	0	1	1	0	0	0
$E4^{(1)}$	0	-1	0	2	-1	0	0	0	0	0	1	0
$E1^{(2)} \leftarrow E1^{(1)}$	1	2	2	-3	1	1	1	-3	-3	0	0	0
$E2^{(2)} \leftarrow E2^{(1)}+E1^{(1)}$	0	-1	2	1	-2	1	0	-3	-3	1	0	0
$E3^{(2)} \leftarrow E3^{(1)}$	0	1	0	-1	1	0	0	1	1	0	0	0
$E4^{(2)} \leftarrow E4^{(1)}+E3^{(1)}$	0	0	0	1	0	0	0	1	1	0	1	0
$E1^{(3)} \leftarrow E1^{(2)}$	1	2	2	-3	1	1	1	-3	-3	0	0	0
$E2^{(3)} \leftarrow E2^{(2)}$	0	-1	2	1	-2	1	0	-3	-3	1	0	0
$E3^{(3)} \leftarrow E3^{(2)} + E2^{(3)}$	0	0	2	0	-1	1	0	-2	-2	1	0	0
$E4^{(3)} \leftarrow E4^{(2)}$	0	0	0	1	0	0	0	1	1	0	1	0
$E1^{(4)} \leftarrow E1^{(3)} + 2\,E2^{(3)}$	1	0	6	-1	-3	3	1	-9	-9	2	0	0
$E2^{(4)} \leftarrow E2^{(3)}$	0	-1	2	1	-2	1	0	-3	-3	1	0	0
$E3^{(4)} \leftarrow E3^{(3)}$	0	0	2	0	-1	1	0	-2	-2	1	0	0
$E4^{(4)} \leftarrow E4^{(3)}$	0	0	0	1	0	0	0	1	1	0	1	0
$E1^{(5)} \leftarrow E1^{(4)} - 3\,E3^{(4)}$	1	0	0	-1	0	0	1	-3	-3	-1	0	0
$E2^{(5)} \leftarrow E2^{(4)} - E3^{(4)}$	0	-1	0	1	-1	0	0	-1	-1	0	0	0
$E3^{(5)} \leftarrow E3^{(4)}$	0	0	2	0	-1	1	0	-2	-2	1	0	0
$E4^{(5)} \leftarrow E4^{(4)}$	0	0	0	1	0	0	0	1	1	0	1	0
$E1^{(6)} \leftarrow E1^{(5)} + E4^{(5)}$	1	0	0	0	0	0	1	-2	-2	-1	1	0
$E2^{(6)} \leftarrow E2^{(5)} - E4^{(5)}$	0	-1	0	0	-1	0	0	-2	-2	0	-1	0
$E3^{(6)} \leftarrow E3^{(5)}$	0	0	2	0	-1	1	0	-2	-2	1	0	0
$E4^{(6)} \leftarrow E4^{(5)}$	0	0	0	1	0	0	0	1	1	0	1	0
$E1^{(7)} \leftarrow E1^{(6)}$	1	0	0	0	0	0	1	-2	-2	-1	1	0
$E2^{(7)} \leftarrow -E2^{(6)}$	0	1	0	0	1	0	0	2	2	0	1	0
$E3^{(7)} \leftarrow E3^{(6)}/2$	0	0	1	0	-0.5	0.5	0	-1	-1	0.5	0	0
$E4^{(7)} \leftarrow E4^{(6)}$	0	0	0	1	0	0	0	1	1	0	1	0

We assert that each equation in the Gauss-Jordan table might have a dimensional interpretation. However, the four equations in a single stage or tableau do not necessarily correspond to a dimensional basis. In the initial tableau, we have four equations representing the standard LTMI basis, while in the final tableau; the four equations follow the $wVA\varepsilon$ basis. In going from the first tableau to the second, the first, third and fourth equations remained intact, and hence they retained their dimensions of L, M, and I in the usual dimensional system. We assigned to the second equation its original value augmented by that of the first equation. This means that in the second tableau, the new second equation is the sum of the second and the first equations in the first tableau, and hence it possesses a new dimension X that is equal to either T or L in a dimensional system, in which T and L are the same. Now, the four equations in the second tableau might be thought to have the dimensions of L, X, M, and I, which belong to different dimensional system, and do not constitute a dimensional basis. Nevertheless, we still observe that there is a dimensional basis in the initial and final tableaus, and that the dimension of any variable is dictated by the column underneath it with respect to the current dimensional basis. For example, the variable I is characterized by the column of exponents $[0\ \ 0\ \ 0\ \ 1]^T$ in the LTMI basis, and by [1 1 0

1]^T in the $wVA\varepsilon$basis. Therefore, we might express its dimension $[I]$ as:

$[I_{cd}]$ = $[L]^0$ $[T]^0$ $[M]^0$ $[I]^1$= $[w]^1$ $[V]^1$ $[A]^0$ $[\varepsilon]^1$ (29)

We note that we have justified our pre-supposition that the dimensional matrix is of rank four. The justification stems from the observation that the Gauss-Jordan procedure was completed without creating an all-0 row. The final result expresses each of the four basis indices a, b, c, and d, in terms of the seven regime indices e, f, g, h, i, j, and k.

$a = -g + 2h + 2i + j - k$ (30a)

$b = -e - 2h - 2i - k$ (30b)

$c = 0.5e - 0.5f + h + i - 0.5j$ (30c)

$d = -h - i - k$ (30d)

There are seven dimensionless products [π] that are shown below via the explicit algebraic expression.

$$\pi = k\, w^a V^b A^c \varepsilon^d E^e d^f \mu^g \rho^h h^i t^j\, I_{cd}^k \quad (31a)$$

$$\pi = k\,(w)^{-g+2h+2i+j-k}(V)^{-e-2h-2i-k} I_{cd}^k (A)^{0.5e-0.5f+h+i-0.5j}(\varepsilon)^{-h-i-k} E^e d^f \mu^g \rho^h h^i t^j I_{cd}^k \quad (31b)$$

$$\pi = k\left(\frac{A^{0.5}E}{V}\right)^e \left(\frac{d}{A^{0.5}}\right)^f \left(\frac{\mu}{w}\right)^g \left(\frac{w^2 A\,\rho}{V^2 \varepsilon}\right)^h \left(\frac{w^2 A\,h}{V^2 \varepsilon}\right)^i \left(\frac{w\,t}{A^{0.5}}\right)^j \left(\frac{I_{cd}}{w\,V\,\varepsilon}\right)^k \quad (31c)$$

$$\pi_1 = \left(\frac{A^{0.5}E}{V}\right) \quad (31d)$$

$$\pi_2 = \left(\frac{d}{A^{0.5}}\right) \quad (31e)$$

$$\pi_3 = \left(\frac{\mu}{w}\right) \quad (31f)$$

$$\pi_4 = \left(\frac{w^2 A\,\rho}{V^2 \varepsilon}\right) \quad (31g)$$

$$\pi_5 = \left(\frac{w^2 A\,h}{V^2 \varepsilon}\right) \quad (31h)$$

$$\pi_6 = \left(\frac{w\,t}{A^{0.5}}\right) \quad (31g)$$

$$\pi_7 = \left(\frac{I_{cd}}{w\,V\,\varepsilon}\right) \quad (31h)$$

We note that the fact that the product $\pi_7 = \left(\frac{I_{cd}}{w\,V\,\varepsilon}\right)$ is dimensionless, was already implied by our earlier observation in (29).

3.2 Using Matrix Method

The dimensional matrix shown below is partitioned into two matrices A and B, the first of which is a full-rank square matrix. Hence this matrix is regular (invertible).

	w	V	A	ε	E	d	M	ρ	H	T	Icd	
L	1	2	2	-3	1	1	1	-3	-3	0	0	0
T	-1	-3	0	4	-3	0	-1	0	0	1	0	0
M	0	1	0	-1	1	0	0	1	1	0	0	0
I	0	-1	0	2	-1	0	0	0	0	0	1	0

The matrix A,its inverse, and the matrix B are

$$A = \begin{bmatrix} 1 & 2 & 2 & -3 \\ -1 & -3 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & -2 & 1 \\ 0 & 0 & 2 & 1 \\ 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 & -3 & -3 & -3 & 0 & 0 \\ -3 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So, the homogeneous matrix equation set by the dimensional matrix is multiplied by the inverse of A to obtain:

$A^{-1} * A$	$A^{-1} * B$	0
		0
		0
		0

	w	V	A	E	E	D	μ	ρ	H	T	Icd	
L	1	0	0	0	0	0	1	-2	-2	-1	1	0
T	0	1	0	0	1	0	0	2	2	0	1	0
M	0	0	1	0	-0.5	0.5	0	-1	-1	0.5	0	0
I	0	0	0	1	0	0	0	1	1	0	1	0

There are seven dimensionless products [π] that are shown below as exponents in the rows of a products matrix. These rows can produce explicit algebraic expressions for the dimensionless products, which are exactly the same as those in (31). Note that the products matrix is partitioned into two matrices, the first of which is the negative transposed of $A^{-1} * B$ above, and the second is a 7 by 7 unit matrix.

	w	V	A	ε	E	D	μ	ρ	h	t	Icd
π1	0	-1	0.5	0	1	0	0	0	0	0	0
π2	0	0	-0.5	0	0	1	0	0	0	0	0
π3	-1	0	0	0	0	0	1	0	0	0	0
π4	2	-2	1	-1	0	0	0	1	0	0	0
π5	2	-2	1	-1	0	0	0	0	1	0	0
π6	1	0	-0.5	0	0	0	0	0	0	1	0
π7	-1	-1	0	-1	0	0	0	0	0	0	1

3.3 The Separation Matrix Method (The Method of Fundamental Modes)

Our last method to find the dimensionless variables is one using Fundamental Modes r Fundamental Solutions. Again, this method assumes prior knowledge of the rank of the dimensional matrix. In our current problem, this rank is assumed by the method to be 4, and hence there are 7 fundamental solutions, called fundamental modes or regimes. For the first fundamental solution, we assume $e = 1$ and $f = g = h = i = j = k = 0$,so that the dimensional matrix equation takes the form $\boldsymbol{x_1} = -\,\boldsymbol{b_1}$, , where matrix $\boldsymbol{A}$ is as given before and vector $\boldsymbol{b_1}$ is the first column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_1}$ is a vector representing the first fundamental-mode value of the basis indices $[a\ b\ c\ d]$T and is given by minus the inverse of $\boldsymbol{A}$multiplied by$\boldsymbol{b_1}$**,** as shown below. When we augment this value of the basis indices by the assumed regime indices ($e = 1$and $f = g = h = i = j = k = 0$), we get the set of indices for π_1, which turns out to be equal to that in Eq. (31d). Computaions for other modes follow similarly and are shown below.

a	b	c	d	$\boldsymbol{b_1}$
1	2	2	-3	1
-1	-3	0	4	-3
0	-1	0	-1	1
0	-1	0	2	-1

$$\boldsymbol{x_1} = -\boldsymbol{A^{-1}} * \boldsymbol{b_1} = \begin{bmatrix} 0 \\ 1 \\ -0.5 \\ 0 \end{bmatrix}$$

a	b	c	D	$\boldsymbol{b_2}$
1	2	2	-3	1
-1	-3	0	4	-1
0	1	0	-1	0
0	-1	0	2	0

$$\boldsymbol{x_2} = -\boldsymbol{A^{-1}} * \boldsymbol{b_2} = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

a	b	c	d	$\boldsymbol{b_3}$
1	2	2	-3	1
-1	-3	0	4	-1
0	1	0	-1	0
0	-1	0	2	0

$$\boldsymbol{x_3} = -\boldsymbol{A^{-1}} * \boldsymbol{b_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

a	b	c	d	$\boldsymbol{b_4}$
1	2	2	-3	-3
-1	-3	0	4	0
0	1	0	-1	1
0	-1	0	2	0

$$\boldsymbol{x_4} = -\boldsymbol{A^{-1}} * \boldsymbol{b_4} = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

a	b	c	d	$\boldsymbol{b_5}$
1	2	2	-3	-3
-1	-3	0	4	0
0	1	0	-1	1
0	-1	0	2	0

$$\boldsymbol{x_5} = -\boldsymbol{A^{-1}} * \boldsymbol{b_5} = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

a	b	c	d	$\boldsymbol{b_6}$
1	2	2	-3	0
-1	-3	0	4	1
0	1	0	-1	0
0	-1	0	2	0

$$\boldsymbol{x_6} = -\boldsymbol{A^{-1}} * \boldsymbol{b_6} = \begin{bmatrix} -1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

a	b	c	d	$\boldsymbol{b_2}$
1	2	2	-3	0
-1	-3	0	4	0
0	1	0	-1	1
0	-1	0	2	-3

$$\boldsymbol{x_7} = -\boldsymbol{A^{-1}} * \boldsymbol{b_7} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Again, there are seven dimensionless products, which exactly replicate the results of the previous two sub-sections.

4. Discussion

All the methods that were calculated in section III have the same result. According to Buckingham theorem, the dimensionless parameters are related in equation (33) :

$$\Phi\ (\pi_1\ , \pi_2 . \pi_3\ , \pi_4\ , \pi_5 , \pi_6\ , \pi_7) = 0 \qquad (32)$$

Finally the mathematical model of the corona discharge current can be stated as:

$$\pi_7\ =\ \Psi\ (\pi_1\ , \pi_2\ , \pi_3\ , \pi_4 , \pi_5\ , \pi_6) \qquad (33)$$

whereΨ is an unknown function, and hence$\left(\frac{I_{cd}}{w\ V\ \varepsilon}\right) = \Psi\ (\left(\frac{EA^{0.5}}{V}\right), \left(\frac{d}{A^{0.5}}\right), \left(\frac{\mu}{w}\right), \left(\frac{w^2\ A\ \rho}{V^2 \varepsilon}\right), \left(\frac{w^2\ A\ h}{V^2 \varepsilon}\right), \left(\frac{w\ t}{A^{0.5}}\right))$ (34)

From equation (34), we can see that the relation between the corona discharge current (I_{cd}) and the wind speed w is a very complex relation, indeed. Basically, there is a direct proportionality between I_{cd} and w within the $\pi_7$regime. However, there is some additional dependence of I_{cd} on w through the unknown function Ψ which is a function of six dimensionless products, four of which have dependence on wind speed. This means that if we can confine the factors influencing I_{cd} to those appearing with it in π_7, then we can take π_7 for a constant k and hence obtain

$$I_{cd} = k\, w\, V\, \varepsilon \sim w \quad (35)$$

Though (35) is an extreme over-simplification, it might serve as a reasonable first engineering cut at the problem that is based solely on dimensional grounds. Its result seems to be in accordance with some previous research, which demonstrates that even a moderate wind speed leads to a significant increase in the corona discharge current[25], [29]. The expected complex effect of wind speed on corona discharge invites further research (both analytical and experimental) to establish a better understanding of the underlying phenomenon. The observation we made in (35) is independent of our partitioning of variables into basis and regime variable, as long as the wind speed is one of the basis variables, and (naturally) the corona discharge current is a regime variable.

5. Conclusion

This paper discusses the dimensional analysis of different phenomena of corona discharges treated in various sources in the open literature. A notable contribution of this paper is to build a larger physical system that involves more variables in the case of the effect of wind speed on the corona discharge current in a lightning rod. By using qualitative analysis based on dimensional grounds, we show that the corona discharge current dependence on wind speed is very complex, indeed. However, under certain simplifying assumptions, we demonstrate that the corona discharge current is directly proportional to wind speed. This purely theoretical work sets the stage for efficient experimental exploration that encompasses all potentially influencing or determining variables.

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