

## ACCURACY OF CYLINDRICAL GEARS

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**Abstract:** At present, in connection with the development of technology and technical, new requirements are imposed on the quality level of transmission mechanisms, in particular, gears. In practice, gears are often designed as theoretical precise gearing, that is, with mating gearing. However, due to errors (manufacturing, installation, due to deformation under load) in real gears, the conjugation conditions are not always met, on the other hand, in some cases, approximate engagement has advantages over conjugated engagement.

**Keywords:** Accuracy of Cylindrical Gears, Gear and Non-Gear Elements

### 1.0 INTRODUCTION

Approximate gears have been mentioned in the literature for a relatively long time, but scientific research on the topic of analysis and synthesis of approximate gears is limited. A significant contribution to the development of the science of gear mechanisms, in the field of manufacturing accuracy, means of control and methods for calculating accuracy was made by the works of Erikhov M.L., Syzrantsev V.N., Sheveleva G.I., Arkhangelsky L.A., Bulgakov E. B., Goldfarba V.I., Dundina N.I., Airapetova E.L., Genkina M.D., Inozemtseva G.G., Kalashnikova N.A., Kane M.M., Kolchina N.I., Kudryavtseva V.N., Kutsokonya V.A., Livshits G.A., Litvina F.L., Popova P.K., Reshetova D.N., Taytsa B.A., Karazina V.I., Balakina P.D., Babicheva D.T., Kryukova V.A., Timofeeva B.P., Tishchenko O.F., Friedlander I.G., Shtripling L.O., Buckingham E., Baxter ML and many other researchers. Standardization of the accuracy of involute cylindrical wheels and gears in Russia is carried out by GOST 1643-81. This standard is basic in relation to the accuracy standards of all other types of gears. GOST 1643-81 carries out the rationing of theoretically conjugated (only involute) gears. The standard specifies tolerances and limit deviations for wheels and gears in one document. At the same time, some of the norms for gears are not directly defined (for example, the kinematic error and side clearance in the gear), but are calculated according to the dependencies established in the standard. The standard contains a number of inaccuracies and contradictions, which is mainly due to its obsolescence.

The practical use of the standard is difficult, since the tolerances and maximum deviations for gears on working axes are indicated. In practice, the values of tolerances and maximum deviations of accuracy standards are taken from the tables of the standard and transferred to the drawing of the wheel. But the fact is that there is no and cannot be a working axis in the wheel drawing, there is only a base axis. ESKD GOST 2.109-73 establishes that in the drawing all dimensions and maximum deviations are indicated relative to the axis in the drawing. The standard is built on functional grounds, the norms of kinematic accuracy, smoothness, contact and side clearance are established. However, these rules are not independent. The degree of accuracy of the wheel and gear of the transmission corresponds to the degree of accuracy of the transmission. However, it is not always possible or technologically justified to make a gear and a wheel of the same degree of accuracy. In this case, the transmission accuracy is not defined. GOST 1643-81 ignores the issue of accuracy of non-toothed transmission elements. Despite the fact that GOST does not define laws or numerical characteristics of the distribution of errors, and does not determine the lower boundary of the dispersion zone of accuracy parameters, many of the above dependencies are built on the assumption of a normal distribution in the course of their addition. In practice, the production technology determines both the lower boundary of the scattering zone (perceived as the lower limit deviation) and the very nature of the distribution. It is worth considering separately the calculation of the bias coefficients. The blocking contour calculation is a method for determining the displacement coefficients exclusively for involute (conjugated) gears, and backlash-free. When designing a transmission, a geometric calculation is made for backlash-free gearing. The introduction of an additional bias in order to obtain a side clearance changes one or another property of the transmission. Thus, it is worth re-checking the transmission quality, taking into account the additional bias. The foregoing confirms the relevance of the problem of analyzing the accuracy parameters of conjugated and approximate gears, their advantages and disadvantages, and the task of

developing and improving methods that allow increasing the level of accuracy of gear mechanisms, in particular, using computer and mathematical modeling. The main results of the research are presented in scientific publications, of which articles are in publications from the list of HAC [1-12], articles in other publications [13-16].

**2.0 METHODOLOGY AND MATERIAL USE**

To solve this task set, the basic provisions of the theory of mechanisms and machines, the theory of higher kinematic pairs, the theory of gearing, matrix methods for the synthesis and analysis of gears, the theory of processing experimental data, as well as the main provisions of theoretical models, methods of mathematical modeling, with further calculation by numerical computer simulation using the software Compass-3D, Mathcad, Matlab/Simulink.

**3.0 FEATURES OF CONJUGATED AND APPROXIMATE TRANSMISSIONS.**

A review of the main types of conjugated and approximate transmissions is carried out. The advantages and disadvantages of involute, cycloidal, adaptive gears and their modifications are analyzed. The principles of rationing the accuracy of gears and gears in the standards are considered. The critical provisions in these Standards are named and substantiated. An analysis of the sources of errors in gears was carried out and ways to improve their accuracy were proposed, as a result of which the research tasks were formulated.

**4.0 CALCULATING THE KINEMATIC ERROR OF SPUR GEARS AND A METHOD OF REDUCING THE KINEMATIC ERROR WHEN ASSEMBLING GEARS.**

All regulatory and technical documentation is based on the summation of the amplitudes of cyclic errors in the course of summing them into a kinematic transmission error. In this case, the initial phases of cyclic errors are completely ignored. It is the consideration of the initial phases during the addition of the harmonic components that makes it possible to achieve the stated result. A calculation method is proposed to reduce the kinematic error of gears with multiple and non-multiple gear ratios. Estimates of the kinematic error and recommendations for its reduction in gear assembly are given. The calculation was carried out for

various transmission options. As a result of the calculation, graphs  $F_i'0$  were obtained depending on the difference in the initial phases  $\epsilon$ . Graphs  $F_i'0$  for transmissions with integer  $u$  can be divided into 3 groups: for  $u = 1; 5; 9$ ,  $u = 2; 4; 6; 8; 10$ ,  $u = 3; 7$ . Graphs in each group are similar to each other, differing only in the value of  $F_i'0$  (Figures 1-3).

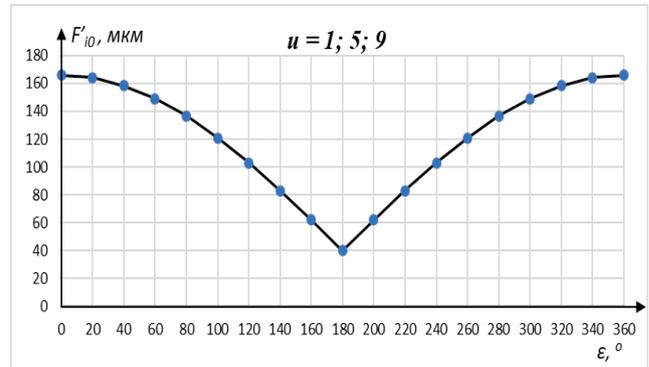


Figure 1. Plots  $F_i'0$  for  $u = 1; 5; 9$

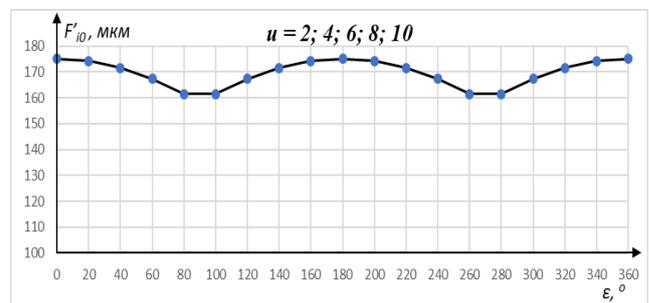


Figure 2 Plot  $F_i'0$  для  $u = 2; 4; 6; 8; 10$

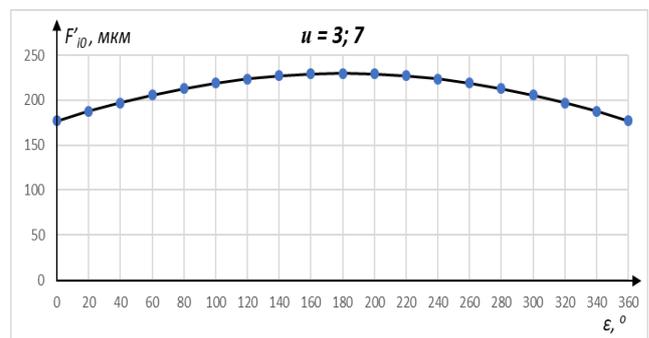


Figure 3. Plots  $F_i'0$  for  $u = 3; 7$

To assess the effectiveness of minimizing the kinematic error when applying this technique, it is proposed to introduce the value of the "exhibition effect" - the ratio of the maximum benefit of minimizing the kinematic error to the maximum possible value of the kinematic error of gears. Using the results of the calculations, you can choose the best assembly option (Tables 1-2).

**Table 1.** Gear alignment effect with  $m = 0.5 \text{ mm}$

$u$	1	2	3	4	5	6	7	8	9	10	
Effect Exhibition%											
<b>0.5 mm</b>											
Degree of Accuracy $k$	3	66.7	5.2	17.4	2.2	7.8	1.1	4.1	0.6	3.0	0.5
	4	66.7	5.7	17.9	2.2	7.8	1.1	4.4	0.7	3.0	0.5
	5	66.7	5.5	17.0	2.2	7.5	1.1	4.3	0.7	2.9	0.5
	6	69.6	5.7	17.6	2.2	7.7	1.1	4.4	0.7	3.0	0.5
	7	71.0	5.9	17.9	2.2	7.8	1.2	4.5	0.7	3.1	0.5
	8	74.4	6.0	18.7	2.3	8.0	1.2	4.6	0.7	3.2	0.5

**Table 2.** Gear alignment effect with  $m = 5 \text{ mm}$

$u$	1	2	3	4	5	6	7	8	9	10	
Effect Exhibition %											
<b>5 mm</b>											
Degree of Accuracy $k$	3	66,7	6,0	20,9	2,5	9,5	1,3	5,6	0,8	3,4	0,6
	4	69,4	6,5	21,8	2,6	9,9	1,4	5,8	0,9	3,5	0,6
	5	74,1	6,7	22,6	2,7	10,1	1,4	5,9	0,9	3,6	0,6
	6	76,2	6,9	23,1	2,8	10,3	1,5	6,0	0,9	3,7	0,6
	7	76,3	6,9	23,1	2,8	10,3	1,5	6,0	0,9	3,6	0,6
	8	75,9	6,9	23,1	2,8	10,3	1,5	6,0	0,9	3,6	0,6

Tables 1 and 2 give an idea of the alignment effect for different tooth modules  $m$ , gear ratios  $u$  and degrees of accuracy  $k$ .

**4.1 METHODS FOR CALCULATING THE SIDE CLEARANCE OF CYLINDRICAL GEARS**

Calculations of the side clearance were carried out using the minimum-maximum method and the probabilistic one (Monte Carlo method). The amplitudes of cyclic errors are considered to be random variables distributed in a given interval in one case evenly, in the other according to the normal law. At the same time, on the basis of practice, the lower limit of the amplitude of cyclic errors has been established, since the standards establish only the upper limit. The initial phases of each cyclic error are considered to be distributed equiprobably in the interval  $0 \div 2\pi$ . The results of calculation by different methods are compared. The calculation was carried out according to the methods: - maximum-minimum; - probabilistic, and the terms are distributed equiprobably (Figure 4a); - probabilistic, and distributed according to the normal law (Figure 4b); As an example, transmissions with  $u = 1 \div 8$  (integer),  $m = 5 \text{ mm}$ ; degree of accuracy 7 C.

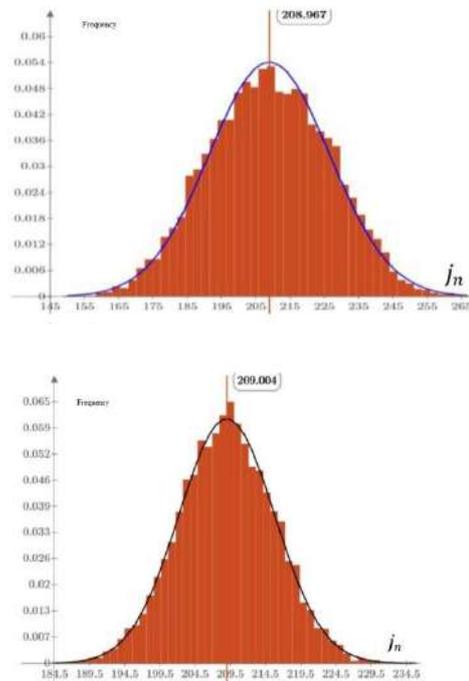
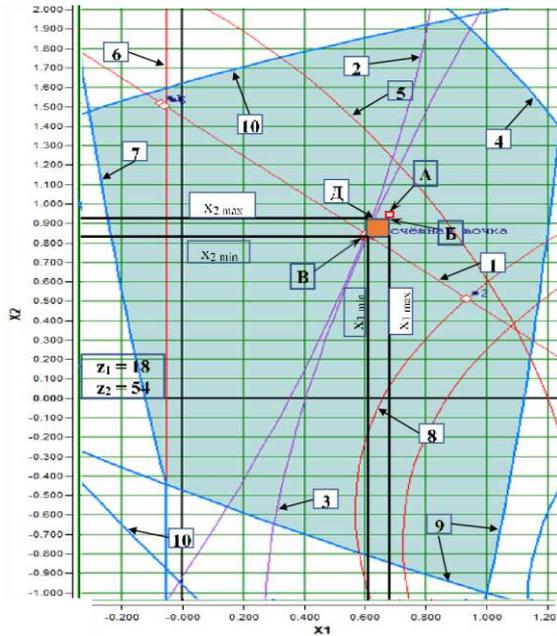


Figure 4. Histogram of calculation by the probabilistic method: a) the terms are distributed equiprobably; b) the terms are distributed according to the normal law



minimum side clearance, bearing in mind the design reference, power and high-speed transmissions, that is, the equal “weight” of criteria in the course of designing critical transmissions is completely unacceptable. The blocking circuit is designed for backlash-free gears. The change in the quality characteristics of transmissions as a result of additional displacements of the original contour is usually neglected. It is shown that this sometimes leads to the exit of a point on the blocking contour with the final (after the introduction of additional illumination) coordinates  $x_1$  and  $x_2$  beyond the limits of unconditional (by interference and overlap coefficient) or conditional (for example, by sharpening of vertices) restrictions. Therefore, it is recommended to check the transmission qualities after choosing an additional offset. From the result of the calculation of the considered cases, it is shown that taking into account the additional displacement reduces the values of  $x$  (for gears with a gap) compared to  $x_i$  (for gapless gears). However, the change in  $x_i^*$  is insignificant; as a result, the calculated values of the point's  $x_1^*$  and  $x_2^*$  are in the allowable area (Figure 5). Methods for assigning an additional displacement of the initial contour and recommendations for choosing the coefficients of displacement of gears are analyzed. Cylindrical gears in Annex 2 Figure 5. Blocking contour for GOST 16532-70 and proposed optimization of the values of the coefficients of the necessary changes (Table 3). Offset.

With the probabilistic method of calculation, a smaller value of the side clearance of the gear was obtained than with the maximum method, which more fully takes into account the specifics of the actual production of gears. The method of calculating the displacement coefficients by the method of multi-criteria optimization of gearing (blocking circuit) using the KOMPAS-3D program is considered. It is indicated that multi-criteria optimization with an equal "weight" of the criteria ignores the functional approach to assigning accuracy parameters, in which the parameters of kinematic accuracy, smoothness, contact and side clearance can be assigned according to different degrees of accuracy, types of interface and

Table 3. Range of allowable values of  $z_2$

$z_1$	8	9	10	11	12	13	14	15	16	17	18	19	20	21
$z_2$ For $x_1 = 0$ $x_2 = 0$	No	No	No	No	No	No	No	$\leq 26$	$\leq 62$	All	All	All	All	All
$z_2$ For $x_1 = 0,3$ $x_2 = -0,3$	No	No	No	No	$\geq 18$	$\geq 19$	$\geq 19$	All	All	All				

**4.2 POSSIBILITIES OF IMPROVING ACCURACY IN REDUCING THE CYCLIC ERROR OF THE TOOTH FREQUENCY OF APPROXIMATE GEARS.**

In this case, approximate transmissions obtained by modifying conjugate transmissions are investigated. In some cases, the mismatch is related to the cutting method that determines the geometry of the engagement. Here, methods for modifying not the

original generating contour, but the finished wheel surface are considered, that is, the deviation of the generating contour along the length and height from the original one is not considered. The deviation of the finished wheel surface from the involute one is considered in order to obtain a localized contact patch. In this case, the goal is obvious - to ensure that the contact does not reach the edge. In this case, we consider as an edge any curve obtained by connecting

(optionally crossing) two surfaces, for example, the involute and transitional surfaces of the tooth. As an example, circular and parabolic modifications for cylindrical wheels are used. The admissible values of the modification parameters are determined taking into account contact deformations. Methods for determining the gear position error are described. Qualitative indicators are compared for circular and parabolic modification of the working surface of one of

the wheels. The engagement overlap coefficient was calculated taking into account contact deformations, i.e. when using the elastic transmission model. The motion parameters for two-pair engagement are determined. In this case, only the profile of one of the wheels of the pair is modified by moving it inward from the original involute profile. The tooth profile of the paired wheel remains unchanged (involute) (Figure 6).

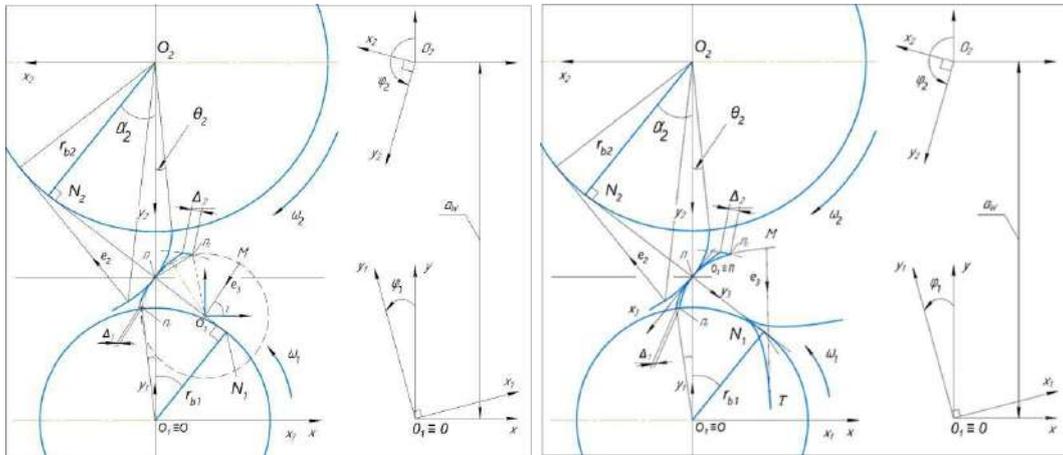


Figure 6. Mathematical model of engagement with circular (a) and parabolic (b) modified profile Based on the contact conditions,

The equations for the coordinates of the contact points of the tooth profiles were compiled. Using the numerical method of the Mathcad software shell, the graphs of the dependences of the motion parameters of the gear links were obtained. With the help of rigid and elastic mesh models, errors of the position function and errors of the gear ratio function are determined (Figures 7 and 8).

Figure 8. Graphs of the error of the position function (a) and the error of the gear ratio function (b) for the circular modification of the deformable mesh model

As a result of modeling, the degree of localization of the contact spot on the tooth profiles was determined, and the increase in the overlap coefficient due to the contact deformation of the teeth was also determined. Also, the characteristics of circular and parabolic modifications were compared; the influence of the gear ratio, the module of the teeth and the magnitude of the modification on the overlap ratio and position errors and gear ratio errors was analyzed. The obtained error values of the gear position function allow us to estimate the level of accuracy of the considered approximate gears in comparison with standard gears according to GOST according to the smoothness and contact standards. By means of graphs of coordinates of the contact points on the tooth profile, you can directly determine the size of the contact patch. Since we consider the contact only along the profile height, the relative size of the contact spot along the tooth height is calculated accordingly, the results are presented in Table 4.

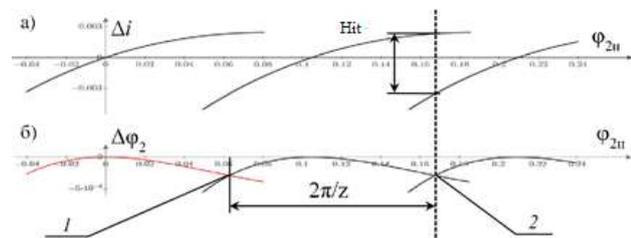
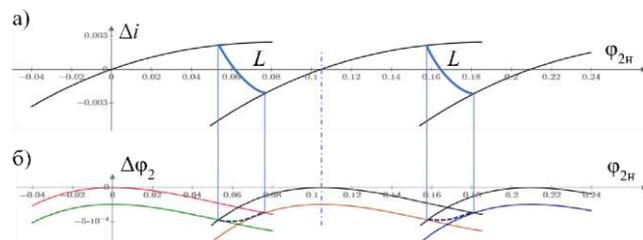


Figure 7. Position error curves (a) and gear ratio (b) of a rigid model for a circular modification, where 1, 2 are reconnection points.



**Table 4.** Comparison of approximate with standard gears

		$f_{z0r, MKM}$			Contact Patch Size, %		
		Estimated	Standard	St of Accuracy	Estimated	Standard	St of Accuracy
Circular modification	Rigid model	23.4	22	7	40.5	40	8
	Elastic model	25.4	28	8	42.5	45	7
Parabolic modification	Rigid model	23.7	22	7	32.2	30	9
	Elastic model	26.3	28	8	38.6	40	8

**4.3 EXPERIMENTAL STUDIES WAS CARRIED OUT IN ORDER TO EXPERIMENTALLY VERIFY THE CALCULATED RESULT**

By minimizing the largest kinematic gear error using the alignment of the wheels during assembly. Experimental studies of the kinematic error of cylindrical gears were carried out by the pulse method using computer tools. To implement the task, an experimental transmission stand was designed, consisting of two gears on parallel axes (Figure 9). The experimental setup was made in the form of a laboratory stand. The main components of the stand, including the gears and the body, were made of plastic materials using rapid prototyping.

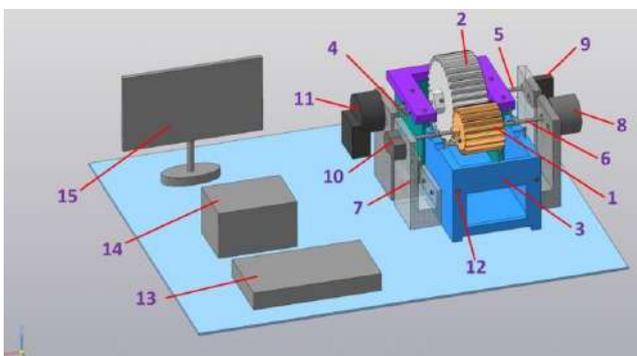


Figure 9. Model of the experimental stand Structural elements are made by 3D printing on the installation "Total Z Anyform L 250G3-2X" and Any cubic Kossel Plus (USA). This unit allows you to make 3D models from ABS, PLC and PLA plastics. Made from ABS plastic. The gears were installed in the experimental stand shown in Figure 10, where 1; 2 – gears, 3 – housing, 8 – stepper motor 28BYJ-48 (China), KR08 (China), 10 – laboratory encoder by Eco Lite Electronics (China), 11

– incremental encoder E6B2-CWZ6C (by Omron, Japan), 16; 17 - rolling bearings.

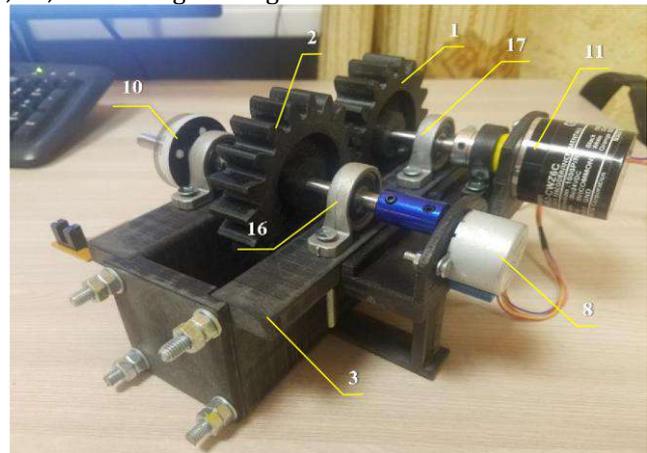


Figure 10. Implemented experimental stand the experimental stand is controlled by a personal computer in real time. MATLAB/Simulink 2019 software packages are used to control the stand and to process the received data. The view of the developed program is shown in Figure 12.

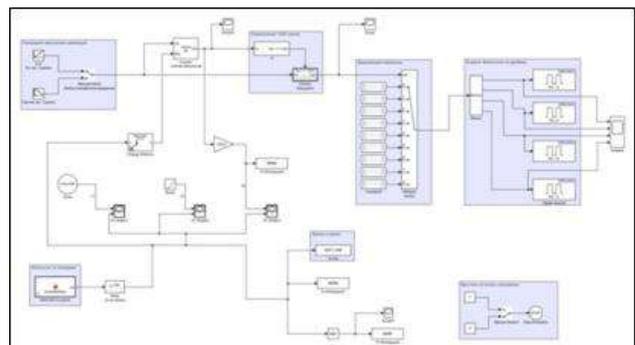


Figure 11. Experimental stand control program the experiment was carried out for gears with gear ratios  $u = 1;3$  and 5. For each gear, 18 experiments were

performed, in each experiment, at the beginning of the experiment; the first tooth of the driving wheel was in contact with the  $n$ th tooth of the driven wheel in succession. In each case, the largest kinematic error is determined. Figure 12 shows the result of the experiment at  $\varepsilon = 180^\circ$ .

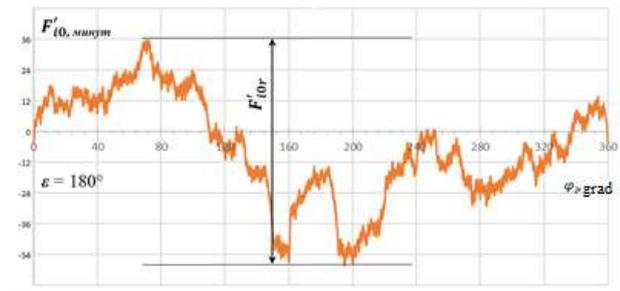


Figure 12. Transfer error function  $u = 1$ ; a)  $\div$  r) correspond to positions  $\varepsilon = 180^\circ$  the displacement function error is determined by the difference between the theoretical and measured values of the displacement function. In each position  $\varepsilon$ , the largest error in the position of the driven wheel is determined. Subsequently, the obtained values of the largest kinematic error for the period (in the case of  $u = 1$ , the period  $T = 2\pi$ ) are transferred to the general graph (Figure 13), where each point corresponds to the value of the largest kinematic error for a given combination of the teeth of the driving and driven wheels, that is, for given ratio of the initial phases of the cyclic error of the reverse frequency  $K1$  and  $K2$ . The experimental graph of the error of the gear position function is compared with the calculated one (Figure 13a).

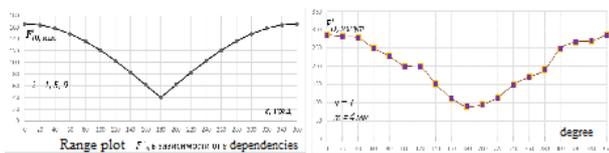


Figure 13. Calculated a) and experimental b) graphs of the position error of the driven wheel for transmission with  $u = 1$  depending on  $\varepsilon$  the experiment is similarly repeated for transmissions with  $u = 3$  and with  $u = 5$  (Figures 14 and 15).

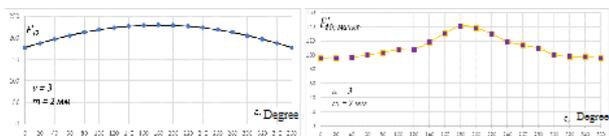


Figure 14. Calculated a) and experimental b) graphs of the position error of the driven wheel for transmission with  $u = 3$  depending on  $\varepsilon$

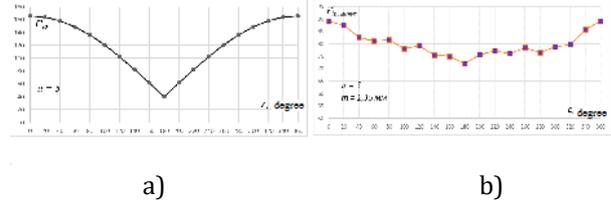


Figure 15. Calculated a) and experimental b) plots of the position error of the driven wheel for transmission with  $u = 5$  depending on  $\varepsilon$  For all the studied transmissions, the magnitude of the experimental exhibition effect was calculated and compared with the calculated value. On the basis of a comparison of the results of theoretical studies with the data of the experiments set in the dissertation, it was found that they are in satisfactory agreement. This indicates the correctness of the calculation methods developed in the dissertation.

5. CONFLICTS OF INTEREST

On behalf of the Authors the corresponding author says there is no conflict of interest

6. RESULTS AND CONCLUSIONS

The degree of reliability of the obtained results is confirmed by the correct use of methods for calculating the error of the position function and the gear ratio function, the theory of mechanisms, methods for analyzing and synthesizing gears, including conjugated and approximate ones, with linear and point contact, contact localization methods to prevent edge contact, and the addition method harmonic functions, the pseudo-optimization method in the Compass program, numerical methods in the Mathcad and MATLAB programs, the main confirmation of the reliability is the results of experiments confirming the theoretical positions and the following conclusions on the work are formulated:

- 1) The critical provisions of the standards are analyzed; their inaccuracies and contradictions are identified. Methods for their refinement are proposed and additional recommendations are given;
- 2) A technique for reducing the kinematic transmission error during assembly is proposed;
- 3) Approximate gearings are analyzed taking into account the influence of elastic deformations on the process of re-conjugation of the teeth, approximate and conjugated (standard) gears are compared and the effectiveness of their application is determined;
- 4) Different approaches to the calculation of the side clearance are compared; the influence of the additional

displacement of the initial contour on the quality of transmissions is estimated;

5) An experimental setup was implemented and the proposed method for improving the kinematic accuracy of gears was tested

## REFERENCES

- [1] Timofeev B.P., Dang N. Increasing the overlap coefficient in the elastic model of the modified gear transmission. *Izvestiyavuzov.Engineering.* - 2019. - No. 10(715). - S. 33-42.
- [2] Timofeev B.P., Dang N. Influence of geometric and kinematic parameters of cylindrical gears on the overlap coefficient during profile modification. - 2019. - No. 10. - S. 29-35.
- [3] Timofeev B.P., Dang N. Minimization of the kinematic error of elementary gears in selective assembly // *Instruments.* - 2019. - No. 6 (228). - S. 37-41.
- [4] Timofeev B.P., Dang N. A method for improving the accuracy of fine-module gears in instrumentation during assembly. *Izvestiyavuzov.Instrumentation.* - 2019. - T. 62. - No. 10. - S. 941-946.
- [5] Timofeev B.P., Dang N. Automation of gear assembly process // *Automation. Modern technologies.* - 2019. - No. 11. - S. 483-487.
- [6] Timofeev B.P., Dang N. Choice of additional displacement coefficients of the initial contour of the gears of cylindrical involute gears // *Heavy Engineering.* - 2019. - No. 10.-S. 44-47.
- [7] Timofeev B.P., Dang N. A new approach to the selection of the displacement coefficients of the initial gear circuit // *Assembly in mechanical engineering, instrumentation.* - 2019. No. 11. - S. 508-512.
- [8] Timofeev B.P., Dang N. Theoretical and practical approach to determining the lateral clearance in gears // *Science-intensive technologies in mechanical engineering.* - 2019. -No. 11(101). - S. 18-23.
- [9] V. M. Medunetsky, N. T. Dang, E. V. Shalobaev, and V. A. Zinkov, "Method of designing and calculating small-sized wheels made of composite materials," *Izvestiyavuzov. Instrumentation.* - 2019. - T. 62. - No. 2. - S. 192194.
- [10] B. P. Timofeev, N. T. Dang, and M. Kh. Chan, "Calculation of side clearance in gears by the Monte Carlo method," *Kontrol. Diagnostics.* - 2020. - T. 23. - No. 7. - S. 42 - 47.
- [11] Timofeev B. P., Dang N. T. Determination of additional displacement coefficients of the initial gear contour using blocking contours // *Handbook. Engineering Journal.* - 2020. -No. 3. - S. 32 - 36.
- [12] Dang N. T. GOST 16532–70: additional calculations are needed // *standards and quality.* -2020. - No. 3 (993). - S. 108-110. - in other publications
- [13] Timofeev B.P., Dang N., Chan M.Kh. Increasing the overlap coefficient of approximate gearing due to contact deformation // *Almanac of Scientific Works of Young Scientists of ITMO University,* 2019. - Volume 2, - P. 76-80.
- [14] Timofeev B.P., Dang N.T., Chan M.Kh. The method of minimizing the kinematic error of gears by optimal alignment of gear wheels during selective assembly // *Modern mechanical engineering: Science and education: materials of the 8th International scientific and practical conference.* Ed. A.N. Evgrafov and A.A. Popovich. - St Petersburg: POLYTEKHPRESS, 2019. - S. 299-311.
- [15] Timofeev B.P., Dang N., Chan M.Kh. Method for improving the accuracy of gears by optimizing the alignment of gears during assembly // *Theory of Mechanisms and Machines.* - 2019. - Volume 2, - S. 92-98.
- [16] Dang N., Medunetsky V.M. Basic technologies for manufacturing gear links for small-sized mechanisms // *Almanac of Scientific Works of Young Scientists of ITMO University.* - 2017. - V. 5. - S. 75-78.