A Study on Dimensional Analysis of Several Corona Discharge Phenomena Using Various Computational Techniques

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Abstract: *Corona discharge occurs when the electric field accelerates charged particles such as electrons and ions. This phenomenon is notorious for its several detrimental effects, but, nevertheless, it is utilized in many beneficial engineering applications. The dimensional-analysis methodology is used herein to deduce the functional dependence of corona discharge on the physical parameters that influence it. This paper investigates and compares four aspects of the corona discharge phenomenon, through utilizing dimensional analysis using three computational methods. These methods are: (a) Gauss Jordan elimination (with row or column interchange, if necessary), (b) the method of the direct transformation matrix, and (c) the method of fundamental modes. All the three methods give the same results of dimensional analysis, albeit each of them has its own stress on certain facets of the analysis. Gauss-Jordan elimination does not assume prior knowledge for the rank of the dimensional matrix, while the direct transformation methods uses intutionistically familiar matrix operations. The method of fundamental modes treats each mode or regime separately and acts as a divide-and-conquer strategy. The general analysis is supplemented with a combinatorial discussion of how to partition the pertinent variables into bases and regimes, and of when a specific such partitioning is possible.*

Keywords: *Dimensional analysis, corona discharge, Gauss Jordan elimination, direct transformation matrix, fundamental modes*

1. Introduction

The corona discharge phenomenon naturally occurs when the electric field accelerates charged particles such as electrons and ions[1]. Specifically, this phenomenon arises as a result of high voltages accompanied by high-speed collision reactions that produce many ions in a gap of space filled with a dielectric such as air or some other insulating gas. Research on corona discharge is interesting not only to assess and mitigate its detrimental effects, but also because this phenomenon can be exploited for many useful applications. Goldman[2] shows that the corona discharge can be applied to electrostatic separation, xerography, optical computation, and disinfection. Besides that, corona discharge can also be applied to separate metals and nonmetals[3], decontamination[4], lightning protection tools such as lightning rods[5],heat transfer enhancement[6-8], seeds treatment[9], and fluidic applications[10].The corona discharge current could be expressed in terms of a general form using dimensional analysis (DA) that specifies the relationship of this current to the pertinent physical quantities that influence it[11]. Information gained through the dimensional analysis of corona discharge is useful to design and conduct experiments to study it. The technique can also help to design a small-scale model or prototype of any potential application system before building a full-size prototype for it.

The basic idea of dimensional analysis was suggested by Euler in 1765 and later by Fourier in the early nineteenth century. In 1877, Lord Rayleigh proposed a novel method of physical dimensions[12]. Then, in 1914, Buckingham introduced the Pi theorem for describing and deriving dimensionless products. Subsequently, the DA literature started to grow dramatically and many applications that depend on dimensional analysis emerged, which utilized results relevant to experimental facts[13]. Many problems can be solved using DA strategies such as modeling virus spread rate[14], derivation of fundamental masses[15], exploration of qualitative physics[16], and assessment of corona discharge[17]. Dimensional Analysis could achieve a reduction of $(n-m)$in the number of the pertinent variables, where n is the number of the initial dimensional variables and m is the number of the final dimensionless products, which act as new variables.

This paper discusses the utilization of dimensional analysis in topics related to corona discharge from four distinct perspectives. These are (a) dimensional analysis of corona discharge current*per se* and how it is influenced by lightning rod tips, (b) dimensional analysis of negative wire to plate corona discharge, (c) dimensional analysis of the breakdown voltage, and (d) dimensional analysis of ozone production by pulsed streamer discharge. The four topics can represent a somewhat comprehensive study on DA of the corona discharge phenomenon where DA on each of these topics will be explained using different methods in order to ensure there are no errors. The methods used will help us to get the appropriate formula for each topic.

Each of the aforementioned problems is investigated via three computational methods. These methods are: (a) Gauss-Jordan elimination (with row or column interchange, if necessary)[14, 15, 18-26], (b) the method of the direct transformation matrix[27], and (c) the method of fundamental modes[21]. All the three methods give the same results of dimensional analysis, albeit each of them has its own stress on certain facets of the analysis. The Gauss-Jordan elimination does not assume prior knowledge for the rank of the dimensional matrix. It integrates the issue of rank determination with that of the solution itself. The direct transformation method might look more conceptually appealing, as it uses intutionistically familiar matrix operations. The method of fundamental modes treats each mode or regime separately and acts as a divide-and-conquer strategy.

2. Dimensional Analysis

This section discusses dimensional analysis for the afore-mentioned four problems related to corona discharge. The analysis will use the afore-mentioned three different methods that are to be explained herein step by step to ensure that the results obtained are without errors, and to provide a useful tutorial exposition.

To apply DA herein, we need to express each physical quantity of interest in corona discharge, inters of basic dimensions, usually those of length (L), time (T), mass (M),and electric current (I).The DA methodology provides correlation between the physical world and mathematics. This physical relation is shown in a simple way via the dimensional matrix in which the columns are the variables and the rows are the basic dimensions[20-26].

Our first computational method uses Gauss Jordan elimination, which is outlined in Fig. 1 for a full-rank dimensional matrix ***D***. This matrix illustrates the dimensions of each physical variable. Here, the full-rank part of the dimensional matrix ***D*** is, initially partitioned into two matrices ***A*** and ***B***, called its basis and regime sub-matrices. These two matrices are subsequently changed via permissible elementary row operations into the identity matrix ***I*** and a matrix ***C***, respectively.We can find the rank of the dimensional matrix from the square matrix ***A*** with the criterion that matrix ***A*** must not be singular, which can be proven by showing that its determinant is not equal to zero. Our second method uses direct transformation of matrices, while and the third method uses fundamental modes, thereby producing the formula$\boldsymbol{C} = \boldsymbol{A}^{-1}\boldsymbol{B}$ and the formula$\boldsymbol{\pi} = -(\boldsymbol{A}^{-1}\boldsymbol{B})^{T}$, respectively[20-26].

Matrix A				*Matrix B*	
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
Matrix I				*Matrix C= $A^{-1}B$*	
1	0	0	0	-	-
0	1	0	0	-	-
0	0	1	0	-	-
0	0	0	1	-	-

Figure 1. An outline of the full-rank part of the dimensional matrix ***D***, (top) initially partitioned into its basis and regime matrices ***A*** and ***B***, and (bottom) finally after implementing Gauss Jordan elimination.

2.1 Lightning rod tips capturing lightning leaders.

The first aspect discussed herein concerns the effectiveness of lightning rod tips in capturing lightning leaders (bidirectional electrically conductive channels of ionized air); a topic that has been studied earlier in Sidik et al[5]. The main thesis of this topic is that an effective lightning rod can be influenced by the shape of the lightning rod tip, which can be a standard pointed shape, concave, blunt (unsharpened), flat, or conical.

This paper shows how dimensional analysis could be used to observe the effect of various lightning rod tips on the corona discharge current. To set the stage for this kind of analysis, we identify the dimensions of a set of six pertinent or relevant variables as shown in Table 1. Later, we partition this set of variables into a set of output or regime variables and another set of basis variables, also called determining or influencing variables. We use each of our three computational methods to obtain dimensionless products where all three methods are found to produce the same results. A dimensionless product formed of the set of six quantities in Table 1 has the general form

$$\pi = k\, {I_{cd}}^{b} \varepsilon^{e} T^{f} A^{g} d^{h} V^{i}, \tag{1}$$

where k is a dimensionless constant, while *b, e, f, g, h, i* are unknown exponents, yet to be inter-related, to be expressed in terms of the basic dimensions of charge (Q), time (T), mass (M), and length (L), as shown in Table 1.

Table 1. Variables for the corona problem

Variable	Symbol	Dimension
corona discharge current	I_{cd}	$T^{-1} Q^{1}$
permittivity	ε	$M^{-1} L^{-3} T^{2} Q^{2}$
time	T	T^{1}
area of the tip	A	L^{2}
distance	d	L
input voltage	V	$M^{1} L^{2} T^{-2} Q^{-1}$

A. Method 1

Table 1 shows that there are six variables and four fundamental dimensions and therefore there are two dimensionless variables if the dimensional matrix has full rank[20-26]. To find these dimensionless products, we use the Gauss-Jordan elimination as follows.

	b	e	f	g	h	i
M	0	-1	0	0	0	1
L	0	-3	0	2	1	2
T	-1	2	1	0	0	-2
Q	1	2	0	0	0	-1
	0	0	0	-2/3	-1/3	1/3
	0	1	0	-2/3	-1/3	-2/3
	-1	0	1	4/3	2/3	-2/3
	1	0	0	4/3	2/3	1/3
	1/2	0	0	0	0	1/2
	1/2	1	0	0	0	-1/2
	-2	0	1	0	0	-1
	3/4	0	0	1	1/2	1/4
I_{cd}	1	0	0	0	0	1
ε	0	1	0	0	0	-1
T	0	0	1	0	0	1
A	0	0	0	1	1/2	-1/2

Each row in the initial and final tableau of the Gauss-Jordan procedure is left labeled by the dimension it represents. Since this procedure did not produce an all-0 row, the dimensional matrix has a full rank of 4, indeed. We can now express each of the first four exponents (called basis exponents) in terms of the remaining two exponents (called regime exponents) as follows

$$b = -i, \tag{2}$$

$$e = i, \tag{3}$$

$$f = -i, \tag{4}$$

$$g = -0.5h + 0.5i. \tag{5}$$

Utilizing equation (1) and equations (2-5), we find the dimensionless product,

$$\pi = k\,(I_{cd})^{-i}(\varepsilon)^{i}(T)^{-i}(A)^{-0.5h+0.5i}(d)^{h}V^{i} \tag{6}$$

$$\pi = k(A^{-0.5}d)^{h}(I_{cd}{}^{-1}\varepsilon T^{-1}A^{0.5}V)^{i} \tag{7}$$

Finally, we can get the two dimensionless productsπ_1 nd π_2,

$$\pi_1 = A^{-0.5}d\,, \tag{8}$$

$$\pi_2 = I_{cd}{}^{-1}\varepsilon T^{-1}A^{0.5}V\,. \tag{9}$$

B. Method 2

In the second method, we employ the direct transformation of the dimensional matrix using the formula: $\boldsymbol{C} = \boldsymbol{A}^{-1}\boldsymbol{B}$, where.

$$\boldsymbol{A}^{-1} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 \\ -3/2 & 1/2 & 0 & 0 \end{bmatrix}, \tag{10}$$

$$\boldsymbol{C} = \boldsymbol{A}^{-1}\boldsymbol{B} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 0 & 1 \\ 1/2 & -1/2 \end{bmatrix}. \tag{11}$$

Following the layout in Fig. 1, we enter the value of $\boldsymbol{C}$ along with a 4 by 4 unit matrix, so as to replace the original dimensional matrix, there by recovering the final tableau above, namely

	b	e	f	g	h	i
I_{cd}	1	0	0	0	0	1
ε	0	1	0	0	0	-1
T	0	0	1	0	0	1
A	0	0	0	1	1/2	-1/2

We obtain the same result for the dimensionless variables π_1 and π_2 as given in equation(8) and (9).

C. Method 3

In this method, we augment the matrix

$$\boldsymbol{\pi} = -(\boldsymbol{A}^{-1}\boldsymbol{B})^{\boldsymbol{T}} = \begin{bmatrix} 0 & 0 & 0 & -1/2 \\ -1 & 1 & -1 & 1/2 \end{bmatrix}, \tag{12}$$

by an appropriate unit matrix of the same row dimension, thereby obtaining the exponents of the dimensionless products, namely

	b	e	f	g	h	i
π_1	0	0	0	-1/2	1	0
π_2	-1	1	-1	1/2	0	1

Based on any of the three methods above, the two dimensionless products can be inter-related by expressing the second of them as an arbitrary function f of the first, i.e., $\pi_2 = f(\pi_1)$,thereby yielding an expression of the corona discharge current in terms of the influencing variables

$$I_{cd}{}^{-1}\,\varepsilon\,T^{-1}\,A^{0.5}\,V = f(A^{-0.5}d), \tag{13}$$

$$I_{cd} = \varepsilon\,A^{0.5}\,V/(T\,f(A^{-0.5}d)). \tag{14}$$

D. Bases, regimes and dimensionless products

There are $\frac{6!}{4!(6-4)!} = 15$ choices partitioning the initial variables into 4 basis variables and 2 regime variables[14, 15, 20-26]. The specific 15 potential sets of pairs of independent regimes obtained are shown in Table 2, which indicates that only 9 of these sets really exist. When possible, we obtain entries in Table 2 either through appropriate manipulation of pair No. 1 (obtained earlier herein) or by using Gauss-Jordan elimination afresh with a rearrangement of the sequence of the initial variables[15].After finding the dimensionless variables, we note that the pairs numbered 8, 9, 11, 13, and 14would have enjoyed the desirable feature of having the corona discharge current as one of the output or regime variables. However, the pairs in lines 8, 11, and 13 are among the 6 pairs that fail to exist because the corresponding inverse matrix $\boldsymbol{A}$ is not invertible, or equivalently because there is no manipulation of π_{a1} and π_{b1} that can produce the purported pair.

Table 2. Pairs of dimensionless products that form independent regimes (a regime variable has power 1 in its own product and power 0 in the other associated product(s)).

No	Set of basis (input) variables	Set of isolated regime (output) variables	π_{ai}	π_{bi}
1	$\{I_{cd}, \varepsilon, T, A\}$	$\{d, V\}$	$\pi_{a1} = \frac{d}{\sqrt{A}}$	$\pi_{b1} = \frac{\varepsilon V \sqrt{A}}{I_{cd} T}$
2	$\{I_{cd}, \varepsilon, T, d\}$	$\{A, V\}$	$\pi_{a2} = \frac{A}{d^2} = \pi_{a1}{}^{-2}$	$\pi_{b2} = \frac{V d \varepsilon}{I_{cd} T} = \pi_{a1}\pi_{b1}$
3	$\{I_{cd}, \varepsilon, T, V\}$	$\{A, d\}$	$\pi_{a3} = \frac{A\varepsilon^2 V^2}{I_{cd}{}^2 T^2} = \pi_{b1}{}^2$	$\pi_{b3} = \pi_{b2}$
4	$\{I_{cd}, \varepsilon, A, d\}$	$\{T, V\}$	-	-
5	$\{I_{cd}, \varepsilon, A, V\}$	$\{T, d\}$	$\pi_{a5} = \frac{I_{cd} T}{\varepsilon V \sqrt{A}} = \frac{1}{\pi_{b1}}$	$\pi_{b5} = \pi_{a1}$
6	$\{I_{cd}, T, A, d\}$	$\{\varepsilon, V\}$	-	-
7	$\{I_{cd}, T, A, V\}$	$\{\varepsilon, d\}$	$\pi_{a7} = \pi_{b1}$	$\pi_{b7} = \pi_{a1}$
8	$\{\varepsilon, T, A, d\}$	$\{I_{cd}, V\}$	-	-
9	$\{\varepsilon, T, A, V\}$	$\{I_{cd}, d\}$	$\pi_{a9} = \pi_{a5}$	$\pi_{b9} = \pi_{a1}$
10	$\{I_{cd}, \varepsilon, d, V\}$	$\{T, A\}$	$\pi_{a10} = \frac{I_{cd} T}{\varepsilon d V} = \frac{1}{\pi_{b2}}$	$\pi_{b10} = \pi_{a2}$
11	$\{T, A, d, V\}$	$\{I_{cd}, \varepsilon\}$	-	-
12	$\{I_{cd}, T, d, V\}$	$\{\varepsilon, A\}$	$\pi_{a12} = \pi_{b2}$	$\pi_{b12} = \pi_{a2}$
13	$\{\varepsilon, A, d, V\}$	$\{I_{cd}, T\}$	-	-
14	$\{\varepsilon, T, d, V\}$	$\{I_{cd}, A\}$	$\pi_{a14} = \pi_{a10}$	$\pi_{b14} = \pi_{a2}$
15	$\{I_{cd}, A, d, V\}$	$\{\varepsilon, T\}$	-	-

2.2 Ozone generation rate of a negative wire to plate corona discharge.

Bo et al.[28, 29] use dimensional analysis to determine the determinants of positive and negative wire-to-plate corona discharge in dry and humid air. The determinants for the negative case are derived from dimensional analysis associated with experimental tests. The dimensional system is obtained from the physical parameters shown in Table 3 which have seven variables. A dimensionless product of the set of these seven quantities in Table 3 is of the form

$$\pi = k\, r_0{}^b a^c V^e d^f V_e{}^g \varepsilon^h \mu^i, \qquad (15)$$

where k is a dimensionless constant, while b, c, e, f, g, h, I are exponents. The quantities of fundamental dimensions are now mass (M), time (T), length (L), and electric current (A).

Table 3. Variables of corona discharge on dry and humid air.

Variable	Symbol	Dimension
ozone generation rate per unit length	r_0	$M^1 L^{-1} T^{-1}$
wire radius	a	L^1
applied voltage	V	$M^1 L^2 T^{-3} A^{-1}$
inter-electrode gap	d	L^1
excess voltage	V_e	$M^1 L^2 T^{-3} A^{-1}$
permittivity	ε	$M^{-1} L^{-3} T^4 A^2$
ion mobility	μ	$M^{-1} T^2 A^1$

A. Method 1

The dimensional matrix involves seven variables and four dimensions, and therefore it produces three dimensionless variables (assumed to be full rank).

	f	g	h	i	b	c	e
M	0	1	-1	-1	1	0	1
L	1	2	-3	0	-1	1	2
T	0	-3	4	2	-1	0	-3
A	0	-1	2	(1)	0	0	-1
	0	0	1	0	1	0	0
	1	(2)	-3	0	-1	1	2
	0	-1	0	0	-1	0	-1
	0	-1	2	1	0	0	-1
	0	0	1	0	1	0	0
	1/2	1	-3/2	0	-1/2	1/2	1
	1/2	0	(-3/2)	0	-3/2	1/2	0
	1/2	0	1/2	1	-1/2	1/2	0
	(1/3)	0	0	0	0	1/3	0
	0	1	0	0	1	0	1
	-1/3	0	1	0	1	-1/3	0
	1/3	0	0	1	-1	1/3	0
d	1	0	0	0	0	1	0
V_e	0	1	0	0	1	0	1
ε	0	0	1	0	1	0	0
μ	0	0	0	1	-1	0	0

Then, we express each of the 4 basis exponents *f, g, h* and *i* interms of the 3 regime exponents c, b and e as follows

$$f = -c\,, \quad (16)$$

$$g = -b - e, \quad (17)$$

$$h = -b, \quad (18)$$

$$i = b\,, \quad (19)$$

and hence, we end up with the 3 regimes or dimensionless products

$$\pi_1 = r^0 \mu \varepsilon^{-1} V_e^{-1}, \quad (20)$$

$$\pi_2 = ad^{-1}, \quad (21)$$

$$\pi_3 = V\, V_e^{-1}. \quad (22)$$

B. Method 2

In method 2 we need to find the matrix $\boldsymbol{C} = \boldsymbol{A^{-1}B}$, where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are the left and right partitions of the initial dimensional matrix. Therefore, we obtain

$$\boldsymbol{A^{-1}} = \begin{bmatrix} 3 & 1 & 2 & -1 \\ 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 1 \end{bmatrix}, \quad (23)$$

$$\boldsymbol{C} = \boldsymbol{A^{-1}B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad (24)$$

and hence we end up with the matrix

	f	g	h	i	b	c	e
d	1	0	0	0	0	1	0
V_e	0	1	0	0	1	0	1
ε	0	0	1	0	1	0	0
μ	0	0	0	1	-1	0	0

which is exactly the same as the final matrix in the Gauss-Jordan elimination.

C. Method 3

$$\boldsymbol{\pi} = -(\boldsymbol{A^{-1}B})^{\boldsymbol{T}} = \begin{bmatrix} 0 & -1 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \quad (25)$$

The dimensional set is:

	f	g	h	i	b	c	e
π_1	0	-1	-1	1	1	0	0
π_2	-1	0	0	0	0	1	0
π_3	0	-1	0	0	0	0	1

The results of methods 1, 2 and 3 show no difference. Then, we use Buckingham's Pi theorem to find the correlation

$$\pi_1 = f(\pi_2 \pi_3), \quad (26)$$

thus,

$$r_0 \mu \varepsilon^{-1} V_e^{-1} = f(ad^{-1} V\, V_e^{-1}), \quad (27)$$

where V_e equal the applied voltage (V) minus the corona inception voltage. Using experimental considerations (outside the scope of dimensional analysis), the unknown function f in (26) is anticipated to be of the following double-monomial form involving three arbitrary constants k_1, k_2, and k_3[29]

$$\pi_1 = k_1 (\pi_2)^{k_2} (\pi_3)^{k_3}, \quad (28)$$

substituting equation (20-22) into equation (28), we obtain

$$r_0 \mu \varepsilon^{-1} (V - V_i)^{-1} = k_1 (ad^{-1})^{k_2} (V(V - V_i)^{-1})^{k_3}. \quad (29)$$

2.3 Breakdown Voltage of Electrode Gaps

Dimensional analysis techniques are used to develop a mathematical equation of the breakdown voltage for different electrode gaps[30]. The distance between the two electrodes is varied in discrete steps to take the values0.5 cm, 0.8 cm, 1.6 cm and 2 cm. Then, the DA technique finds a correlation of the breakdown voltage with certain influencing physical parameters. The total set of eight basis and regime variables of the breakdown voltage phenomenon are listed together with their dimensions in Table 4. The variable N_e is usually called the number of electrons, but it actually denotes the total charge of such electrons. The variable S denoting solar radiation depicts the time-rate area-density of solar energy, and hence has the dimension

$$[S] = [Energy]\, L^{-2}\, T^{-1} = MLT^{-2} L\, L^{-2}\, T^{-1} = MT^{-3} \quad (30)$$

Piah et al.[30] use an AC step-up voltage transformer of 240 V: 100 kV for the voltage breakdown experiment. Based on their paper[30] the peak voltage of 90 kV can already cause corona discharge with the length of the corona channel being 4 cm. The form of a dimensionless product of the set of eight variables in Table 4 is

$$\pi = k\, V_b{}^a\, d^b\, S^c\, A_{eff}{}^e\, p^f\, h^g\, N_e{}^i\, R_a{}^j\,, \quad (31)$$

Where k is a dimensionless constant, while *a, b, c, e, f, g, i,*and *j* are so far unknown exponents, that are yet to be partially inter-related.

Table 4. Variables of the breakdown voltage phenomenon.

Variable	Symbol	Dimension
breakdown voltage	V_b	$M L^2 T^{-2} Q^{-1}$
electrode gap length	d	L
solar radiation	S	$M T^{-3}$
effective area	A_{eff}	L^2
pressure	p	$M L^{-1} T^{-2}$
humidity (vapor density)	h	$M L^{-3}$
number of electrons	N_e	Q
surface roughness	R_a	L

A. Method 1

The positions of the matrices ***A***, ***B***, ***C*** and ***I*** are the same as those in subsection 2.2. The system has eight variables and four dimensions, which mean that there are $8-4=4$ dimensionless variables (assuming a rank of 4 for the dimensional matrix).

	f	g	i	j	a	b	c	e
M	(1)	1	0	0	1	0	1	0
L	-1	-3	0	1	2	1	0	2
T	-2	0	0	0	-2	0	-3	0
Q	0	0	1	0	-1	0	0	0
	1	1	0	0	1	0	1	0
	0	(-2)	0	1	3	1	1	2
	0	2	0	0	0	0	-1	0
	0	0	1	0	-1	0	0	0
	1	0	0	1/2	5/2	1/2	3/2	1
	0	1	0	-1/2	-3/2	-1/2	-1/2	-1
	0	0	0	-1/2	3	1	0	2
	0	0	1	0	-1	0	0	0
	1	0	0	1/2	5/2	1/3	3/2	1
	0	1	0	-1/2	-3/2	-1/2	-1/2	-1
	0	0	(3)	1	0	1	0	2
	0	0	1	0	-1	0	0	0
	1	0	0	1/2	5/2	1/2	3/2	1
	0	1	0	-1/2	-3/2	-1/2	-1/2	-1
	0	0	1	1/3	0	1/3	0	2/3
	0	0	0	(-1/3)	-1	-1/3	0	-2/3
p	1	0	0	0	1	0	3/2	0
h	0	1	0	0	0	0	-1/2	0
N_e	0	0	1	0	-1	0	0	0
R_a	0	0	0	1	3	1	0	2

From the final matrix we get:

$$f = -a - 1.5c\,, \qquad (32)$$

$$g = 0.5c\,, \qquad (33)$$

$$i = a\,, \qquad (34)$$

$$j = -3a - b - 2e\,. \qquad (35)$$

Therefore we can write:

$$\pi_1 = V_b\, N_e\, p^{-1}\, R_a^{-3} \qquad (36)$$

$$\pi_2 = d\, R_a^{-1} \qquad (37)$$

$$\pi_3 = S\, h^{0.5}\, p^{-1.5} \qquad (38)$$

$$\pi_4 = A_{eff}\, R_a^{-2} \qquad (39)$$

B. Method 2

Method 2 uses the direct transformation matrix as follows:

$$A^{-1} = \begin{bmatrix} 0 & 0 & -1/2 & 0 \\ 1 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 \end{bmatrix}, \qquad (40)$$

$$C = A^{-1}B = \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 0 & -1/2 & 0 \\ -1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 2 \end{bmatrix}. \qquad (41)$$

The result of augmentation of the ***C*** matrix by a unit 4 by 4 matrix is:

	f	g	i	j	a	b	d	e
p	1	0	0	0	1	0	3/2	0
h	0	1	0	0	0	0	-1/2	0
N_e	0	0	1	0	-1	0	0	0
R_a	0	0	0	1	3	1	0	2

The result for the set of dimensionlessproducts is the same as the one obtained in method 1.

C. Method 3

In method 3 we get:

$$\pi = -(A^{-1}B)^T = \begin{bmatrix} -1 & 0 & 1 & -3 \\ 0 & 0 & 0 & -1 \\ -3/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}. \qquad (42)$$

The dimensional set is:

	f	g	i	j	a	b	d	e
π_1	-1	0	1	-3	1	0	0	0
π_2	0	0	0	-1	0	1	0	0
π_3	-3/2	1/2	0	0	0	0	1	0
π_4	0	0	0	-2	0	0	0	1

There are no differences in the results among the three methods 1, 2 and 3. The dimensionless products can be inter-related via:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4). \qquad (43)$$

Hence, the breakdown voltage is expressed as

$$V_b = N_e^{-1} p^1 R_a^{3} f\left(d\, R_a^{-1}, S\, h^{1/2}\, p^{-1/2}, A_{eff}\, R_a^{-2}\right) \qquad (44)$$

2.4 Ozone production by pulsed streamer discharge in oxygen.

Buntat et al.[31] use dimensional analysis to determine the effect of charge configuration parameters and electrical discharge on ozone production. The electrical discharge associated with an electric field in a dielectric separating two conductors could generate ozone. The model of ozone production is developed with dimensional analysis to deduce the correlation among the various physical parameters. We study ozone production by using eight variables, with one variable among them denoting the corona inception voltage V_o. Based on Buckingham theorem, the form of each dimensionless product of the set of eight variables in Table 5 is

$$\pi = k\, O_3{}^{a}\, f_r{}^{b}\, V^{c}\, \tau^{e}\, \varepsilon_r{}^{g}\, d_g{}^{h}\, P^{i}\, f^{j}, \tag{45}$$

where k is a dimensionless constant, while *a, b, c, e, g ,h, i,* and *j* are exponents. The ozone production yield denotes the mass of ozone released per input energy to the gas.

Table 5. Variables of the ozone production problem.

Variable	Symbol	Dimension
ozone production yield	o_3	T^2L^{-2}
gas flow rate	f_r	L^3T^{-1}
peak pulse voltage	V	$L^2M^1T^{-3}A^{-1}$
pulse duration	τ	T^1
relative permittivity	E_r	$T^4A^2L^{-3}M^{-1}$
gap length	d_g	L^1
pressure	P	$M^1L^{-1}T^{-2}$
pulse repetition frequency	f	T^{-1}

A. Method 1

In the first method, we use eight variables and four dimensions. So, we get four dimensionless variables. (assuming full rank for the dimensional matrix). The Gauss Jordan procedure goes as follows

	g	h	i	j	a	b	c	e
L	(-3)	1	-1	0	-2	3	2	0
M	-1	0	1	0	0	0	1	0
T	4	0	-2	-1	2	-1	-3	1
A	2	0	0	0	0	0	-1	0
	1	(-1/3)	1/3	0	2/3	-1	-2/3	0
	0	-1/3	4/3	0	2/3	-1	1/3	0
	0	4/3	-10/3	-1	-2/3	3	-1/3	1
	0	2/3	-2/3	0	-4/3	2	1/3	0
	1	0	-1	0	0	0	-1	0
	0	1	-4	0	-2	3	-1	0
	0	0	(2)	-1	2	-1	1	1
	0	0	2	0	0	0	1	0
	1	0	0	-1/2	1	-1/2	-1/2	-1/2
	0	1	0	-2	2	1	1	2
	0	0	1	(-1/2)	1	-1/2	1/2	1/2
	0	0	0	1	-2	1	0	-1
ε_r	1	0	0	0	0	0	-1/2	0
d_g	0	1	0	0	-2	3	1	0
p	0	0	1	0	0	0	1/2	0
f	0	0	0	1	-2	1	0	-1

The final matrix equation is equivalent to the four scalar equations

$$g = 0.5c\,, \tag{46}$$

$$h = 2a - 3b - c\,, \tag{47}$$

$$i = -0.5c\,, \tag{48}$$

$$j = 2a - b + e\,. \tag{49}$$

Then, we can deduce the following four dimensionless products:

$$\pi_1 = O_3 d_g{}^2 f^2\,, \tag{50}$$

$$\pi_2 = f_r d_g{}^{-3} f^{-1}\,, \tag{51}$$

$$\pi_3 = V d_g{}^{-1} \varepsilon_r{}^{1/2} P^{-1/2}, \tag{52}$$

$$\pi_4 = \tau f\,. \tag{53}$$

B. Method 2

In method 2, we use a direct transformation matrix:

$$\boldsymbol{A^{-1}} = \begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1/2 \\ 0 & -2 & -1 & 1 \end{bmatrix}, \tag{54}$$

$$\boldsymbol{C} = \boldsymbol{A^{-1}B} = \begin{bmatrix} 0 & 0 & -1/2 & 0 \\ -2 & 3 & 1 & 0 \\ 0 & 0 & 1/2 & 0 \\ -2 & 1 & 0 & -1 \end{bmatrix}. \tag{55}$$

The dimensional matrix is:

	g	h	i	j	a	b	c	e
ε_r	1	0	0	0	0	0	-1/2	0
d_g	0	1	0	0	-2	3	1	0
P	0	0	1	0	0	0	1/2	0
f	0	0	0	1	-2	1	0	-1

This result has no difference with the result of method 1.

C. Method 3

In method 3, we get:

$$\boldsymbol{\pi} = -(\boldsymbol{A^{-1}B})^{\boldsymbol{T}} = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 0 & -3 & 0 & -1 \\ \frac{1}{2} & -1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{56}$$

The dimensional set is:

	g	h	i	j	a	b	c	e
π_1	0	2	0	2	1	0	0	0
π_2	0	-3	0	-1	0	1	0	0
π_3	1/2	-1	-1/2	0	0	0	1	0
π_4	0	0	0	1	0	0	0	1

We can see that the result is the same as those of the other methods. According to Buckingham's theorem, the dimensionless parameters can be written as:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4), \qquad (57)$$

An empirical equation for ozone production assumes that the unknown function f in (57) is simply of a triple-monomial form, characterized by the four dimensionless constants $Dc, \varepsilon_2, \varepsilon_3,$ andε_4.

$$\pi_1 = Dc\,(\pi_2)^{\varepsilon_2}(\pi_3)^{\varepsilon_3}(\pi_4)^{\varepsilon_4}. \qquad (58)$$

Substituting equation (50-53) into equation (58), we get the following expression for ozone production

$$O_3 = Dc\left({d_g}^2 f^2\right)\left(f_r {d_g}^{-3} f^{-1}\right)^{\varepsilon_2} \left(V {d_g}^{-1} {\varepsilon_r}^{\frac{1}{2}} P^{-\frac{1}{2}}\right)^{\varepsilon_3} (\tau f)^{\varepsilon_4} (59)$$

where equation (59) is the same as the result in Buntat et al[27].

3. Conclusions

The corona discharge phenomenon could be explored in many areas which could be of benefit for enhancing the human quality of life. The characteristics of various variants of this phenomenon are investigated herein through the use of dimensional analysis, which is capable of reducing the number of variables and finding correlations among the pertinent physical parameters, and hence it serves as a very useful prelude to the conduction of an actual field experiment.

The present paper has several important contributions. The dimensional analysis technique is applied herein successfully to four distinct aspects of corona discharge, with an aim of distinguishing differences in influencing factors under different conditions. The same results are obtained with three equivalent computational methods for handling the dimensional matrix, which are explained in ample detail in this paper. A major novel contribution of the paper is its combinatorial scheme of partitioning pertinent variables into basis variables and regime variables. The paper demonstrates that occasionally desired sets of regime variables might fail to exist.

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