Compressed Sensing: Non-uniform Sampling and Random Sampling for Electrocardiogram Signal Reconstruction

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Abstract: *The possibility to reconstruct spares signal from fewer linear measurements is known as compressed sensing (CS). Researchers show that it's possible to reconstruct signals and images via compressed sensing methods using a few numbers of measurements. Electrocardiography (ECG) is a diagnostic tool that measures and records the electrical activity of the heart. In this work we intended to use compressed sensing algorithms to reconstruct the ECG signal using a few numbers of measurements, which facilitate signal archiving, speed up signal transmission and reduce energy consumption. Two signals (simulated and real signals) have been reconstructed by using random sampling and non-uniform sampling via ℓ1-norm. The result shows that the signal recovered perfectly using only the halt of the data without affecting the signal quality. The performance of the resulting signal is assessed by experts besides metric performance measures such as cross-correlation and percentage mean square difference.*

Keywords: *Compressed Sensing, ECG, ℓ1-norm, Random Sampling, Non-uniform Sampling, Reconstruction.*

1. Introduction

The acquisition of electrical activity of the heart is captured over time using an external electrode attached to the patient skin known as Electrocardiography [1]. ECG Is a useful tool to examine the electrical and muscular function of the heart, which can be detected and amplified by the ECG machine. It gives information about heart rate, rhythm, and electrical activity [2]. Cardiac muscles are the same as others; contract in response to electrical depolarization of the muscle cells. The first wave in the ECG signal is called the P wave made by the right and left atria or upper chambers following a flat line when the electrical impulse goes to the bottom chambers. The next wave is a QRS complex generated by the right and left bottom chambers or ventricles. And the last wave is a T wave that represents electrical recovery or returns to a resting state for the ventricles.

The standard ECG signal was recorded using 12 different recording leads. Six of these are recorded from the chest and four are recorded from the limps. To get correct ECG records it's essential to place the ten records in the correct position during signal recording. By using ECG it's possible to find out if parts of the heart are too large or are overworked, besides determining how long the electrical wave takes to pass through the heart.

ECG signal is widely used for clinical diagnosis of heart disease because it's a non-invasive way. To detect information from heart signals very long-term records were required [3]. In this case, much data from patients are collected thus compression is essential for reducing energy consumption, storage space, and transmission time. Several methods have been addressed for ECG signal compression [4, 5, 6]; such as wavelet compression methods, transform methods, direct methods, and other compression methods [7, 8]. Transform methods based on compressed sensing methods have been used to reconstruct the ECG signal from a few numbers of measurements because of their simplicity and high performance. During the reconstruction, it's important to make sure that the resulting signal has been reconstructed perfectly with sufficient quality to ensure avoidance of misdiagnosis. During selecting the amount of data for signal reconstruction it's important to select the number of measurements that sufficient to reconstruct the signal with high quality, epically in ECG signal to ensure the reconstruction of the resulting signal with high performance to avoid miss diagnosis.

CS is a novel method for signal and image reconstruction using a few numbers of samples less than that proposed by traditional sampling theory [9, 10, 11, 12]. Several methods for recovering a sparse signal from a limited number of measurements have been proposed [13, 14]. CS theory recently has shown great interest in signal and image processing because of its potential enable to the reconstruction of signals and images using fewer numbers of measurements than that suggested by sampling theory. Compressed sensing has been applied in various areas of medical signal image processing such as [15, 16, 17]. In CS a set of linear measurements together with a non-linear recovery process were used. To work with a few numbers of measurements, compressed sensing theory requires the sensed signal to be sparse on a given orthogonal basis and the sensing vectors to be incoherent with this basis [18]. CS is mainly concerned

with low coherence pairs. The incoherence properties hold for many pairs of bases, including, for example, delta spikes and the sin waves of a Fourier basis, or the Fourier basis and wavelets significantly, this incoherence also holds with a high probability between an arbitrarily fixed base and randomly generated one.

In this work, we intend to apply compressed sensing methods ($\ell 1$-norm algorithm), which are not commonly used in practice to a simulated ECG signal filtered with four different types of filters and a real ECG signal to reconstruct the signal by minimizing the amount of data used for reconstruction while conserving the quality

2. Material and Methods

2.1 Principle of Compressive Sensing

New methods for signals and image reconstruction has been proposed using low numbers of measurements a few years ago, the proposed algorithm is known as compressed sensing [12]. Compressed sensing was proposed by [12] and has been used in wide areas and for different applications such as its application in Magnetic resonance imaging (MRI), computed tomography (CT), Ultrasound RF echo, and Doppler ultrasound signal [13]. The basic principle of CS refers to the possibility of reconstructing signals which are sparse on certain bases from a lower number of projections on a set of random vectors [10]. By using CS it's possible to get an answer to the question of how to achieve efficient compression.

In its standard form, for the 1D case, the theory of CS considers signals x belonging to RN which are K-sparse on some basis $\Psi = \{\Psi i, i, 1 \ldots, N\}$ they can write as $x = \Psi\alpha$ where the element of $\alpha = \{\alpha i, i = 1, \ldots, N\}$. In CS theory it has been proved that such a signal can be recovered from a few numbers of order $M = O(K\log(N/K))$ of non-adaptive projections on a set of random vectors with an error of the same order as the error obtained by truncating the N-K non-significant coefficients to zero [12].

For sparse signals, instead of acquiring N samples according to the sampling theorem, a smaller number M of signal independent projections on random vectors is taken, from which the signal can be rebuilt. The M×1 projection vector y can be written

$$y = \Phi x + n = \Phi\Psi\,\alpha + n = \Theta\alpha + n \qquad \text{or}$$

$$y = \Phi f = \Phi\Psi\hat{x}$$

This means that the sample y of the signal f is a linear function of *f*. The sensing matrix Φ is a M x N matrix where M << N, implying that sampling and compression are now performed in one step. So, y becomes a M x 1 vector, while f is N x 1. Due to the sparsity-inducing matrix Ψ the vector $\hat{x}$ is k-sparse, meaning that it has at most k non-zero entries [19]. So this leads to the possibility of reconstruction of signals f and y even though y has fewer samples than the signal sampled at the Nyquist rate. The effective sampling rate has thus lowered.

2.2 ECG Signal

An electrocardiogram is one of the most important diagnostic tools used in the field of medicine. The recorded waveform helped in the diagnosis of the patient heart condition which ranged from minor to life-threatening. ECG signal is generated by a nerve impulse stimulus. The generated voltage over the service of the body ranged from μV to a few mV. When electrodes are attached to the skin they sense the electrical current and transmit them to an electrocardiograph. The electrical current transform into the waveform, which represents the heart depolarization-repolarization cycle [20]. Therefore any change in heart rhythm caused by cardiac arrhythmias will reflect in the person's ECG also. This makes an ECG signal a useful tool used around the world by a physician to analyze heart conditions. The electrical activity of the heart is obtained by using 12 electrodes placed at designated locations in the human body [21].

The normal ECG signal for one cardiac cycle is presented in figure (1). One cycle of ECG signal in general consists of a P wave, a QRS complex, a T wave, and U wave. The isoelectric line also known as baseline voltage is considered the line tracking from the T wave to the next P wave [21].

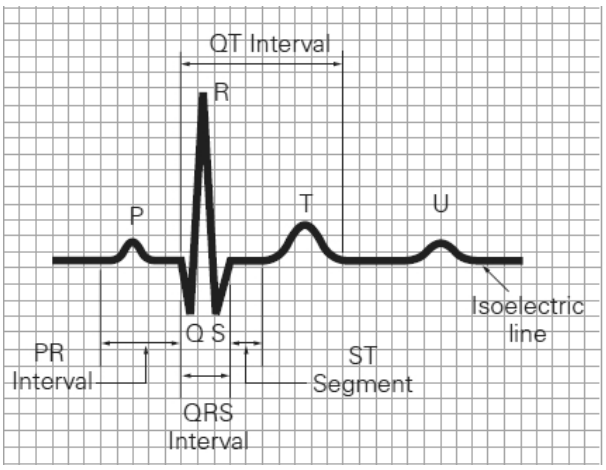

Fig -1: Normal ECG signal for one cardiac cycle

ECG signals are affected by different kinds of noise during their acquisition and transmission. The resulting noise corrupts the resulting signal and hence analysis of the ECG signal becomes very difficult. The most common types of ECG signal noise are, muscles

artifact, electrode motion, baseline wander and channel noise [22]. For perfect ECG signal diagnosis, it's important to collect many cycles, which can be achieved by collecting and storing much data. So applying CS for ECG signal will be effective and reduce the amount of data needed to reconstruct the signal.

2.3 ECG Signal Reconstruction

The workflow for ECG signal reconstruction and evaluation is presented in figure (2). This work aims to reconstruct the ECG signal using a different number of measurements and four different types of filters, then evaluate and compare the result.

For this experiment, two types of ECG signals were used. The first is a simulated signal generated with MATLAB using a sampling frequency of 500. The second is an original signal from HIT-BIH open repository database. Four filtering techniques (Sgolay filter, moving average filter, moving weight filter, and smooth filter) were used to remove unwanted noise components from the signal before and after reconstruction, to get a signal with good performance (high SNR and low PRD). The recovered signal was evaluated by using experts, cross-correlation, and percentage mean square difference.

Simulated ECG signals generation, signal reconstruction, filtering, and signal evaluation were performed in MATLAB software.

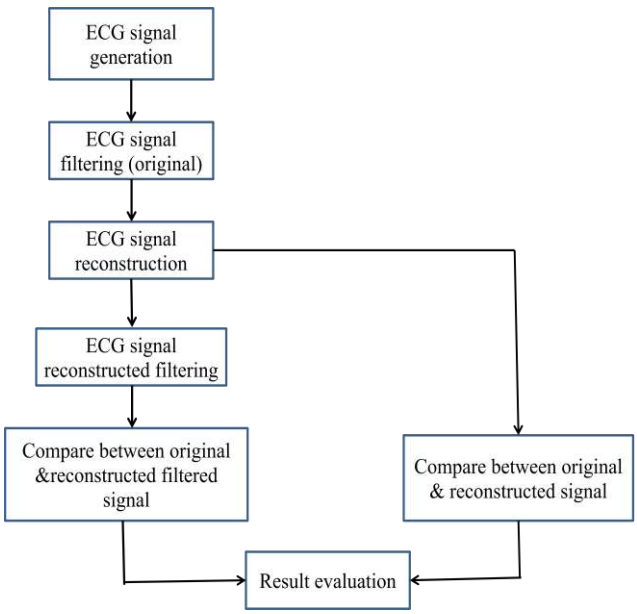

Fig -2: Block diagram of the complete process

2.4 ECG Signal Reconstruction

The original and simulated signal filtered by using the four types of filters mentioned above then applied the CS algorithm ($\ell 1$-norm algorithm) to the resulting signal. The signal was reconstructed by reducing the number of measurements to two third and a half of the total number of measurements. After that the recovered signal is compared with the original signal and also the recovered signal is filtered and compared with the original one. The final signals were evaluated by using cross-correlation and percentage mean square difference.

3. Result and Discussion

Simulated and original ECG signals were reconstructed using a compressed sensing algorithm after removing noises by using four different types of filters. Figures 4 - 9 show the reconstruction result of ECG signal using four different filters and different numbers of measurements, the quality of the reconstructed signal improved as the numbers of measurements increased and according to the filter type. Figure 3 shows the result of the simulated signal filtered with four filters used in this work.

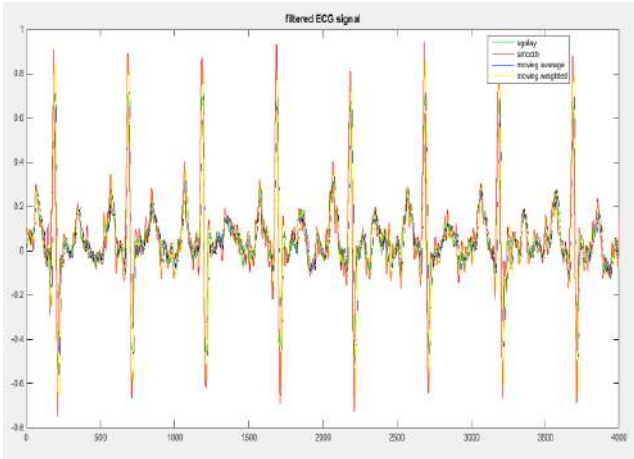

Fig -3: Filtered signal with four different filters

Figure 4 shows a comparison between original signals filtered with a moving weighted filter and reconstructed signal. The result shows that using one-third of the data gives a reconstructed signal with acceptable performance for both visual inspection and reconstruction error, the quality of the reconstructed signal increased as the number of reconstruction samples increased. The percentage mean square difference calculated from the original and reconstructed signal was (0.49).

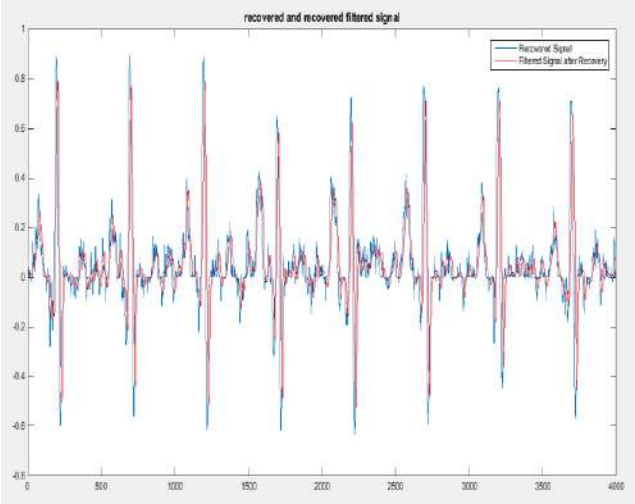

Fig -4: Original signal and recovered signal (using moving weighted filter)

Figure 5 shows the original signal filtered with a moving average filter and reconstructed signal using one-third of the number of measurements. The resulting signal was evaluated visually and metrically, and the result showed unacceptable performance with a percentage mean square difference of (1.36). An increasing number of measurements to half of the data gives high performance and lowers the error, but this lead to increasing storage space.

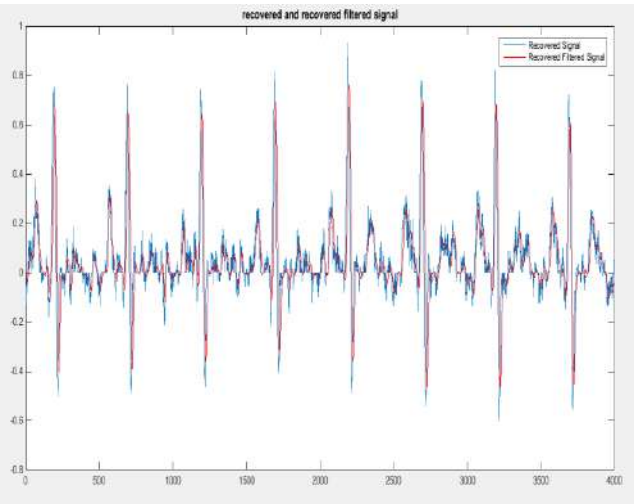

Fig -5: Original signal and recovered signal (using moving average filter)

Figure 6 shows the original signal filtered with a smooth filter and recovered signal using one-third of the data. The signal was recovered with a quality lower than that obtained when the signal was filtered with a moving weighted filter then recovered and higher than that obtained with the signal filtered with a Sgolay filter. The percentage mean square difference calculated was (0.82).

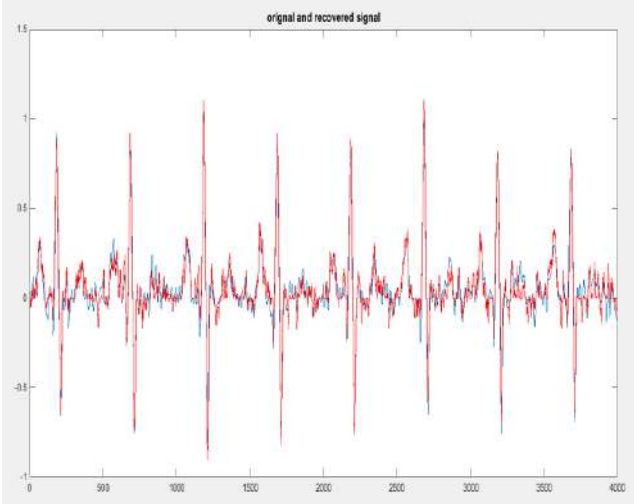

Fig -6: Original signal and recovered signal (using a smooth filter)

Figure 7 shows the original signal filtered with a Sgolay filter and reconstructed signal using one-third number of the measurements. The result shows that the signal was reconstructed with a good performance but the quality of the resulting signal less than that obtain by the signal filtered with a moving weighted filter and smooth filter and then recovered. The percentage mean square difference calculated for the resulting signal was (1.98).

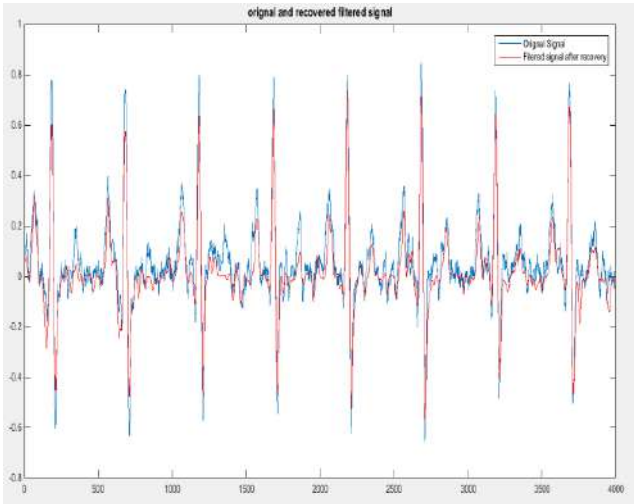

Fig -7: Original signal and recovered signal (using Sgolay filter)

Besides the simulated signal an original ECG signal was used and recovered using compressed sensing figure, 8 represents the original signal and figure 9 shows the reconstructed signal. The result shows that using half number of measurements can reconstruct the signal with good performance and without loss of any important details, increasing the number of measurements can increase the quality of the recovered signal but this can increase the storage space. Also, the signal was evaluated visually by an

expert specialist he indicate that the performance of the signal was perfect and there is no loss of details which is lead to a faulty diagnosis.

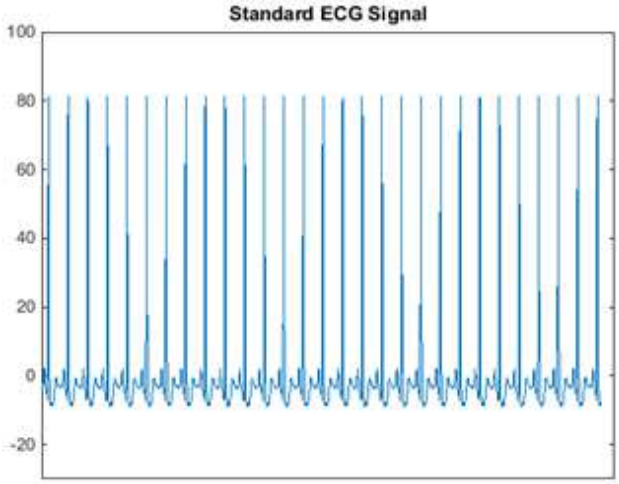

Fig -8: Real ECG signal

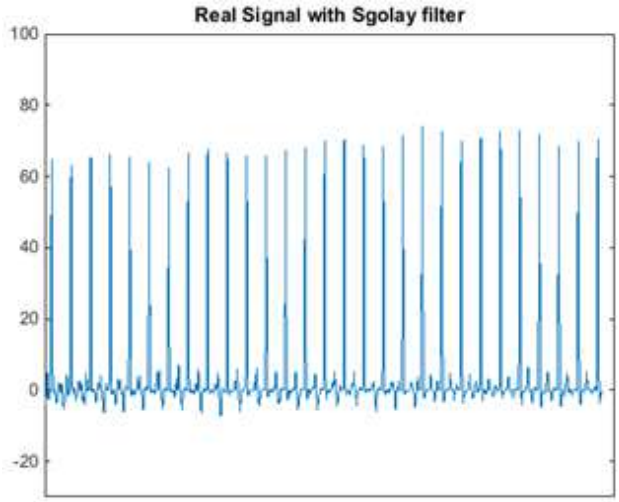

Fig -9: Real ECG signal recovered using compressed sensing

The maximum normalized cross-correlation for the simulated signal between signals was performed as follows:

The cross-correlation between the original signal filtered with the four mentioned filters and recovered signal, the signal filtered with smooth filter and then recovered gives the highest correlation (0.965) and the lowest result was (0.94) from the Sgolay filter. In the cross-correlation between the original signal and the filtered signal after recovery with the four filters, the recovered signal filtered with moving weighted gives the highest correlation (0.99) and the smooth filter gives the lowest value (0.89). In the cross-correlation between the original and filtered signal before reconstruction using the four filters, the Sgolay filter gives the highest correlation (0.98) and the smooth filter gives the lowest value (0.90). From the overall correlation shown in table (1), the highest correlation was obtained from the reconstructed signal and then filtered by moving weighted filter (0.99). The experts also indicate that the filtered signal with a moving weighted filter after reconstruction gives a high-performance signal.

The reconstruction time was calculated for the signal the result shows that the time increased as the number of reconstruction samples increased, and a lower time was obtained from a reconstructed signal filtered with a moving average filter (1.4 seconds) when one-third of measurements were used for all types of filters. The reconstruction time for the original signal was 6.74 seconds and this time is acceptable for a real-time ECG signal.

Table 1: Reconstruction time and Maximum normalized cross-correlation (MNCC) at zero lag for simulated ECG signal filtered with four different filters

MNCC	*Sgolay*	*Moving average*	*Moving weighted*	*Smooth*
Original & Re	0.94	0.965	0.96	0.965
Filtered after	0.965	0.98	0.99	0.889
Filtered	0.98	0.96	0.976	0.90
Re. Time	1.98	1.4	1.94	1.91

Table (2) shows the cross-correlation % for the simulated signals filtered with four filters, the result shows that the signal.

Table 2: Cross-correlation % for simulated ECG signal filtered with four different filters:

MNCC	*Sgolay*	*Average*	*Weighted*	*Smooth*
Recovered	94	96.5	96	96.5
Filtered after	96.5	98	99	88.9
Filtered before	98	96	97.6	90

The cross-correlation for the original signal was calculated the result shows that the recovered signal filtered with a moving weighted filter gives a higher result (0.958). Table 3 shows the cross-correlation for the original ECG signal. Also, the table indicates the importance of filtering the signal during the reconstruction process.

Table 3: Result of cross correlation between the original signal and reconstructed signal for real ECG signal

Records	*MNCC*	*Correlation %*
Original & not filtered	0.897	89.7
Original & filtered Sgolay	0.926	92.6
Original & filtered smooth	0.952	95.2
Original & filtered average	0.935	93.5
Original & filtered	0.958	95.8

4. Conclusions

This paper presents an ECG signal compression based on a compressed sensing algorithm after filtering. Random matrix projection with normal distribution is

used for applying compressed sensing to ECG signal. Applying compressed sensing to signals leads to decreasing the lenses of data to less than half in this work to one-third, this is helping in saving the storage space. The experimental results show that compressed sensing leads to significant data reduction and reconstructed the ECG signal with high performance using a few numbers of measurements. We compared the quality of the reconstructed signal visually by expertise and performance metric. We found that using one-third of the original data can reconstruct original and simulated ECG signal filtered with four different types of filters with good quality. Percentage mean square difference calculated from the original signal filtered with four types of filters and reconstructed signal, the percentage mean square difference from signal filtered with moving weighted filter was (0.49) and the highest value was (1.98) from Sgolay filter. The best-normalized cross-correlation was obtained from the reconstructed signal and then filtered by moving weighted filter (0.99). The study confirms that the ECG signal can be reconstructed with compressed sensing which leads to saving the storage space and give areal time-signal with a good performance.

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