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Novel Model of Atomic Spacetime Based on Atomic Astring Metriant Functions

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Abstract: *A novel theory of spacetime atomization/ quantization based on Atomic AString Functions is described. Using Atomic Series and Atomization Theorems allows representing polynomials, analytic functions, and solutions of differential equations including General Relativity via the superposition of finite Atomic Functions resembling flexible quanta, metriants, or elementary distortions adjusting their locations to reproduce different metrics. The theory may uphold some A. Einstein's aspirations of a new "atomic theory" envisaged in 1933based on the Atomic Function sevolving since the 1970s.*

Keywords: *Spacetime, quantum, atomic function, AString, soliton, metriant, lattice*

1. ATOMIC ASTRING FUNCTIONS AND ATOMIC SPACETIME MODEL – BRIEF HISTORY

The theory of *Atomic Spacetime* evolving since 2017 [1]-[5]is based on extensions of the theory of Atomic and AString Functions [6]-[14]known since the 1970s towards General Relativity and spacetime physics [12]-[22]. Notably, in a 1933lecture [15], A. Einstein challenging probabilistic quantum mechanics envisaged a new kind of "atomic theory" based on "mathematically simplest concepts and the link between them" to solve some "stumbling blocks" of continuous field theories to describe quantized fields. Interestingly, some of Einstein's aspirations of a novel "atomic theory" can be realized with the theory of Atomic Functions (AF) pioneered in the 1970s by V.L. Rvachev and V.A. Rvachev [6]-[12] without connections to Einstein's theories until 2017-2022 author's works on Atomic AString Functions and Atomic Solitons in spacetime physics [1]-[5].

1.1 Atomic Functions

The theory of Atomic Functions (AF) [1]-[14] has been evolving since 1967-1971 when V.L. Rvachev[1] and V.A. Rvachev discovered and researched a finite pulse function $up(x)$ for which derivativespulses would conveniently be similar to the original pulse shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1) for|x| \le 1,$$
$$up(x) = 0 \; for|x| > 1. \quad (1.1)$$

This and other similar functions possess unique properties of infinite differentiability, smoothness, nonlinearity, nonanalyticity, finiteness, and compact support like widely-used splines. Importantly, other functions like polynomials, sinusoids, exponents, and other analytic functions can be represented via a converging series of shifts and stretches of AFs. So, like from 'mathematical atoms' [6]-[14], smooth functions can be composed of the AF superpositions, and those 'atoms' have been called Atomic Functions in the 1970s.

As per a historical survey [10], while some representations of Atomic Functions have been known since the 1920s, the systematic foundation of AF theory has been developed by V.L Rvachev and V.A. Rvachev [6]-[8] and enriched by many followers from different countries [9]-[14],[23]-[30] notably by schools of V.F. Kravchenko [9]-[12], B. Gotovac, H. Gotovac [27],[28], and the author [1]-[5],[23]-[25], with the number of papers and books observed in [10] has grown to a few hundred.

1.2 AString Functions and Atomic Solitons

In 2017, the author noted [1]-[5] that AF $up(x)$ (1.1) is a composite object consisting of two kink functions called AStrings [1]-[5] making them more generic:

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \quad (1.2)$$

Moreover, AString (1.2) is not only a 'composing branch' but also an integral of $up(x)$. AString derived from the theory of Atomic Functions is related to the Fabius function [30] known since the 1970s. Composing AF pulse (1.1) via kink-antikink pair (1.2) of nonlinear AStrings resembles 'solitonic atoms' (or bions) from the theory of soliton dislocations [2],[4],[31],[32,[33]. This led to the theory of Atomic

[1]*Vladimir Logvinovich Rvachev (1926-2005), https://en.wikipedia.org/wiki/Vladimir_Rvachev, Academician of National Academy of Sciences of Ukraine, author of 600 papers, 18 books, mentor of 80 PhDs, 20 Doctors and Professors including the author.*

Solitons [2],[4] where AString (1.2) becomes a solitonic kink while $up(x)$ is a 'solitonic atom' made of AStrings.

1.3 Atomic Spacetime model brief history

The ability of AFs and AStrings to compose polynomials and analytic functions from 'mathematical atoms' has led to the 2017 intuition idea [3] that curved spacetime emerging from Einstein's GR theory[15]-[21]can also be presented as a superposition of overlapping finite 'solitonic atoms'. It resembles the 'spacetime quantization' ideas of composing a curved spacetime from elementary finite 'building blocks' which may also contribute to field theories [34]-[41]where fields and particles are interpreted as some 'excitations of spacetime' [18],[36].AString (1.2) cannot only compose curves from an expansion of finite kinks but also compose 'finite atoms' (1.1) from kink-antikink AString pairs. It means that both spacetime expansion and matter/fields can be interpreted as a complex network of overlapping AStrings resembling strings from string theories [41] presumed to be the 'building blocks' of all fields. So, validated by physicists the purely mathematical Atomic Spacetime theorymay be important for some modern field theories [37]-[44].

1.4 Atomization theorems

Atomic Spacetime theory has found further support in 2022 with *Atomization Theorems*[1],[14]formalizing how polynomials, analytic functions, and solutions of differential equations[1]-[22]as complex as Einstein's GR equations can be presented via superpositions of shifts and stretches of multi-dimensional Atomic AString Functions. Noted 'preservation of analyticity' for Ricci tensor [1],[14]allows not only representing spacetime field via superposition of some finite splines but also deriving the concrete expression for those splines via Atomic Function as a finite nonanalytic function capable of exactly representing a polynomial of any order, hence analytic fields appearing in many theories of mathematical physics.

2. DERIVING ASTRING METRIANT FUNCTION

Let's consider an introductory problem of composing a straight x and curved $\tilde{x}(x)$ spacelines via superposition of some finite *metriant functions* [1]$m(x), x \in [-1, 1]$:

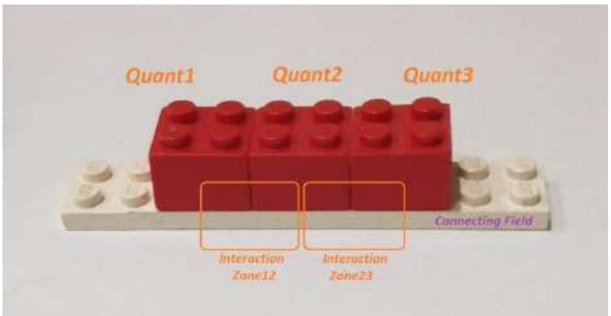
$$x = \sum_{k=-\infty}^{\infty} am\left(\frac{x-ka}{a}\right);$$

$$\tilde{x}(x) = \sum_{k=-\infty}^{\infty} c_k m\left(\frac{x-b_k}{a_k}\right) \qquad (2.1)$$

Composing a spaceline from elementary Lego-like pieces set at regular points ka resembling quanta of width $2a$(Fig. 1). We seek spaceline x to appear not only as a translation (2.1) but also in 'interaction zones' between quanta ($a = 1$) (Fig.1, a, b):

$$x \equiv \dots m(x-1) + m(x) + m(x+1) + \cdots;$$

$$x \equiv m\left(x - \tfrac{1}{2}\right) + m\left(x + \tfrac{1}{2}\right), x \in \left[-\tfrac{1}{2}, \tfrac{1}{2}\right]. \qquad (2.2)$$



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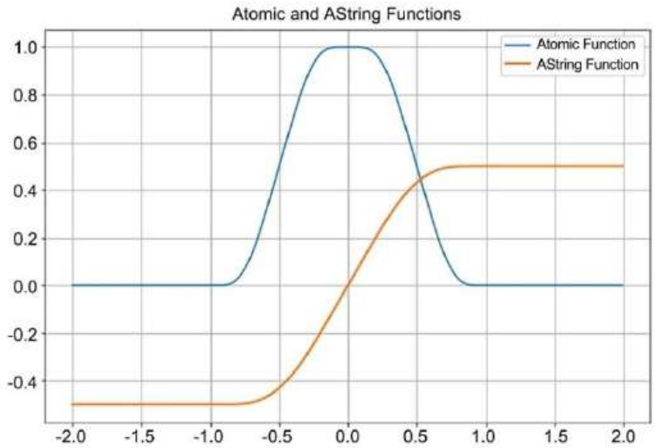

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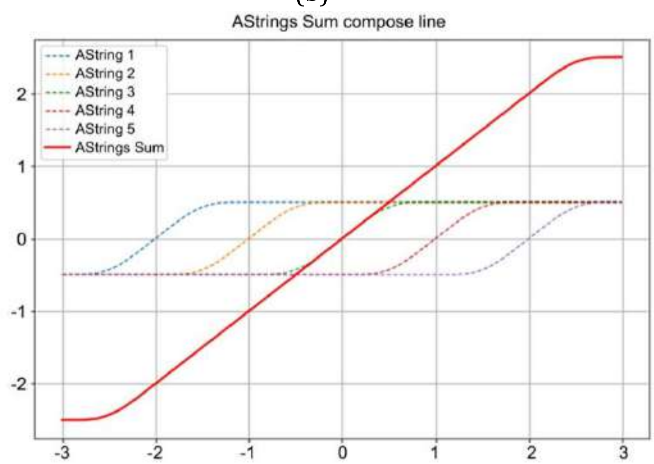

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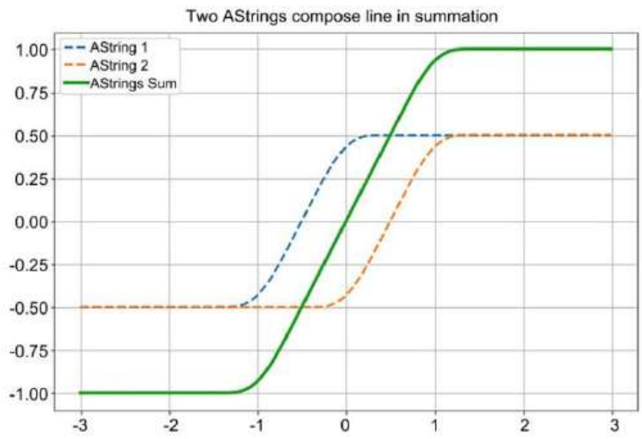

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Fig-1: (a) Lego model with interaction zones; (b) Desired metriant function and its derivative; (c) Expansion of space by the sum of metriant functions; (d) Emergence of line $y = x$ by summing two metriant functions in "interaction zone".

It can be achieved with widely used polynomial splines [23,24] but it leads to a 'polynomial trap' problem [19] imposing artificial polynomial order on spacetime models and not being able to compose a smooth curve $\tilde{x}(x)$(2.1) of *arbitrary* polynomial order. For example, quadratic splines can exactly reproduce only parabolic shapes, which is not sufficient. Instead, seeking a solution amongst finite functions for which derivatives are expressed via themselves

$$p'(x) = f(p(x)) = cp(ax+b) + dp(ax-b)(2.4)$$

and demanding smooth connections between nodes [1],[14]yields the so-called atomic function (AF) $up(x)$[1]-[12] discovered in the 1970s by V.L. Rvachev and V.A. Rvachev [6] (Fig.1, b)

$$up'(x) = 2up(2x+1) - 2up(2x-1),$$
$$p(x) = up(x). \qquad (2.5)$$

The desired metriant function $m(x)$(2.1) would be the integral of $up(x)$ called *AString* [1]-[5]:

$$p(x) = up(x), m(x) = \int_0^x up(x)dx = AString(x),$$
$$x \equiv \sum_k AString(x-k). \qquad (2.6)$$

AString shaped as a kink (Fig.1, b) can compose both straight and curved lines from solitary pieces offering spacetime quantization models based on Atomic and AString Functions [1]-[5],[14] described hereafter.

3. ATOMIC AND ASTRING FUNCTIONS

Let's describe Atomic [1]-[12] and AString [2]-[5] Functions in more detail.

3.1. Atomic Function

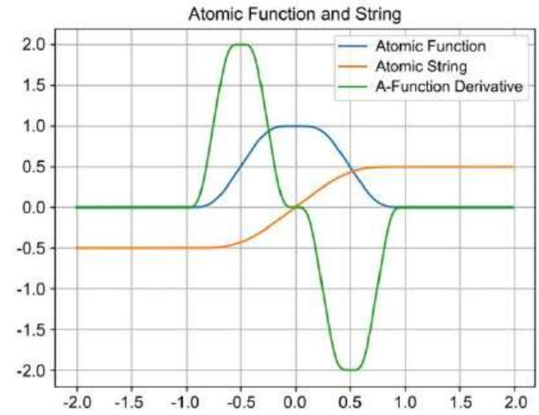
Atomic Function (AF) (V.L. Rvachev, V.A. Rvachev, [6], 1971) $up(x)$ is a finite compactly supported non-analytic infinitely differentiable function (Fig.2) with the first derivative expressible via the function itself shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1) for|x| \leq 1,$$
$$up(x) = 0\ for|x| > 1. \qquad (3.1)$$

With exact Fourier series representation [1]-[12]

$$up(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\sin(t2^{-k})}{t2^{-k}} dt,$$
$$\int_{-1}^{1} up(x)dx = 1, \qquad (3.2)$$

the values of $up(x)$ can be calculated with available computer scripts [2],[4],[9]-[12],[25].



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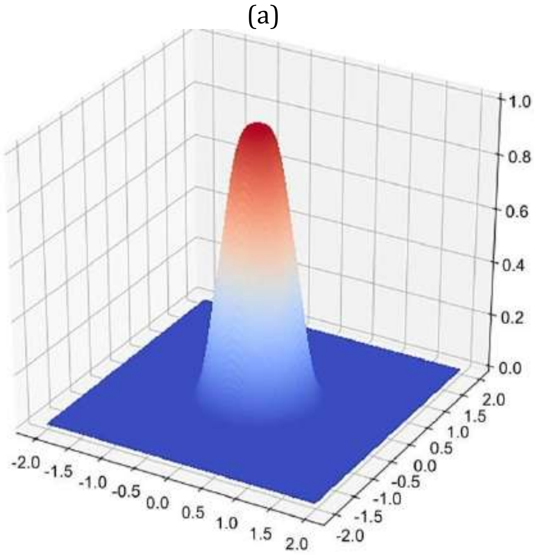

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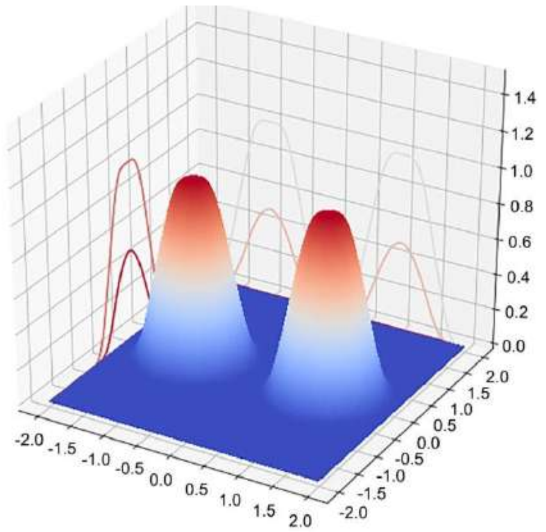

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Fig-2: (a) Atomic Function pulse with its derivative and integral (AString) (b) Atomic Function pulse ("solitonic atom") in 2D (c) Two Atomic Function pulses ("solitonic atoms" or "atomic solitons").

Higher derivatives $up^{(n)}$ and integrals I_m can also be expressed via $up(x)$ [6-12, 27]

$$up^{(n)}(x) = 2^{\frac{n(n+1)}{2}} \sum_{k=1}^{2^n} \delta_k \, up(2^n x + 2^n + 1 - 2k),$$

$$\delta_{2k} = -\delta_k, \delta_{2k-1} = \delta_k, \delta_1 = 1;$$

$$I_m(x) = 2^{C_m^2} up(2^{-m} x - 1 + 2^{-m}), x \le 1;$$

$$I_m(x) = 2^{C_m^2} up(2^{-m+1} - 1) + \frac{(x-1)^{m-1}}{(m-1)!}, x > 1;$$

$$I_1(x) = up(2^{-1} x - 2^{-1}); \; I_1{}'(x) = up(x). \quad (3.3)$$

AF satisfies *partition of unity*[1]-[12] to exactly represent the number 1 by summing up individual overlapping pulses set at regular points ... -2, -1, 0, 1, 2… (Fig.3, a):

$$\ldots up(x-2) + up(x-1) + up(x) + up(x+1) + up(x+2) + \cdots \equiv 1. \quad (3.4)$$

This property is related to the following double symmetry [1]-[12]:

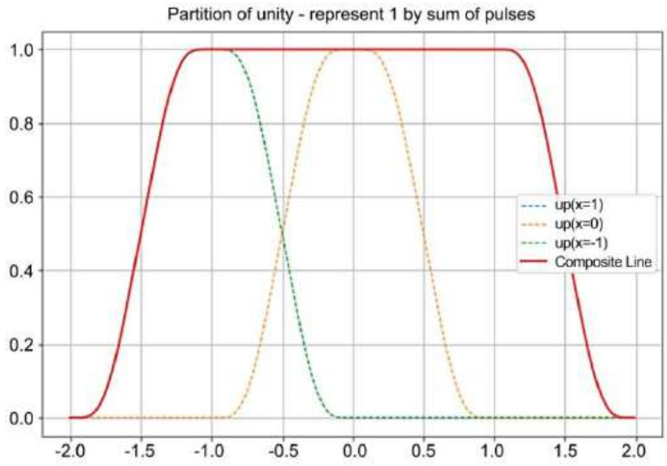
$$up(x) = up(-x), x \in [-1, 1];$$

$$up(x) + up(1-x) = 1, x \in [0, 1]. \quad (3.5)$$

Generic AF pulse of width $2a$, height c, and center positions b, d has the form

$$up(x, a, b, c, d = 0) = d + c * up((x-b)/a).$$

$$\int_{-a}^{a} c\,up(x/a)dx = ca. \quad (3.6)$$



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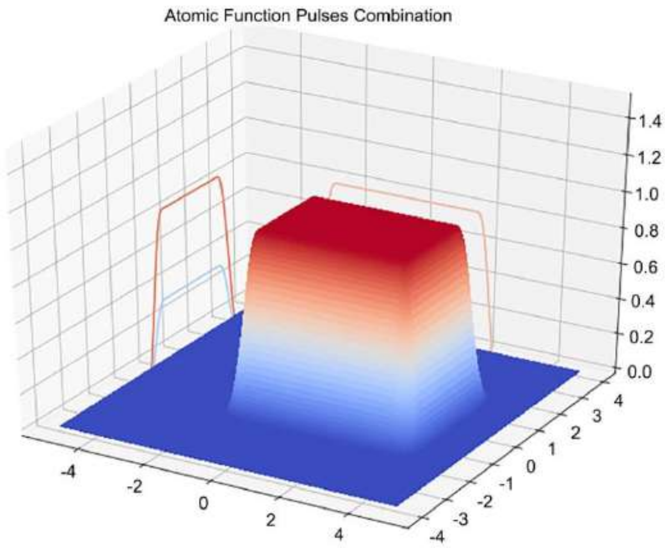

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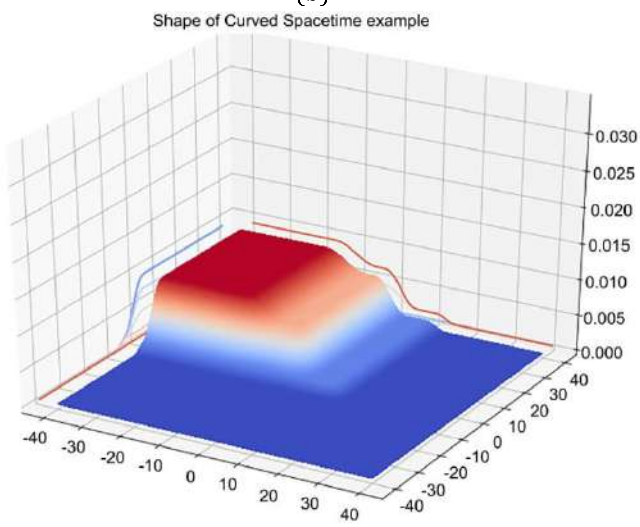

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Fig-3: (a) Partition of unity with Atomic Functions; (b) Representation of flat surface via summation of AFs; (c) Curved surface as a superposition of "solitonic atoms".

Multi-dimensional atomic functions [1]-[12],[29] (Figs.2,3) can be constructed as either multiplications or radial atomic functions:

$$up(x, y, z) = up(x)up(y)up(z),$$

$$up(r) = up(\sqrt{x^2 + y^2 + z^2}),$$

$$\iiint cup\left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}\right) dxdydz = ca^3. \quad (3.7)$$

3.2. AString Function

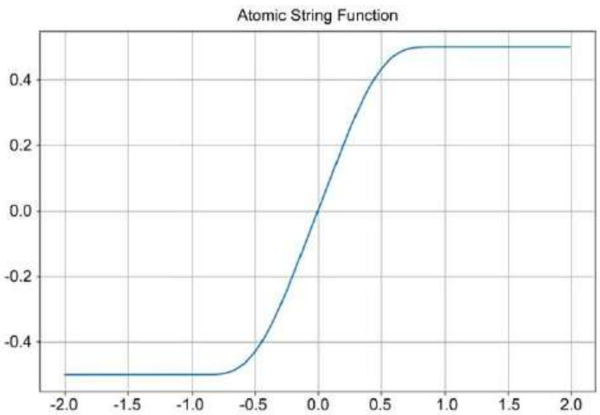
AString function (Fig.4) was introduced in 2018 by the author [1]-[5] as an integral (3.3) and 'composing branch' of $up(x)$:

$$AString^{'}(x) = AString(2x+1) - AString(2x-1) = up(x). \quad (3.8)$$

AString has a form of a solitary kink (Fig.4, a) which can compose a straight line $y = x$both between and as a translation of AString kinks leading to spacetime atomization/quantization ideas (§2,6):

$$x \equiv AString\left(x - \frac{1}{2}\right) + AString\left(x + \frac{1}{2}\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right];$$

$$x \equiv \ \ldots\ AString(x-2) +$$
$$AString(x-1) + AString(x) +$$
$$AString(x+1) + AString(x+2) \ldots \quad (3.9)$$



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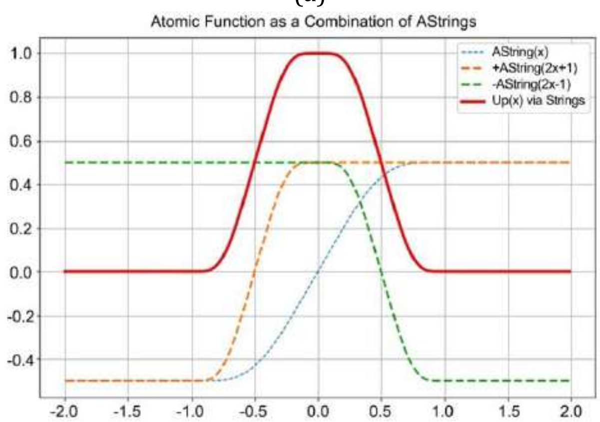

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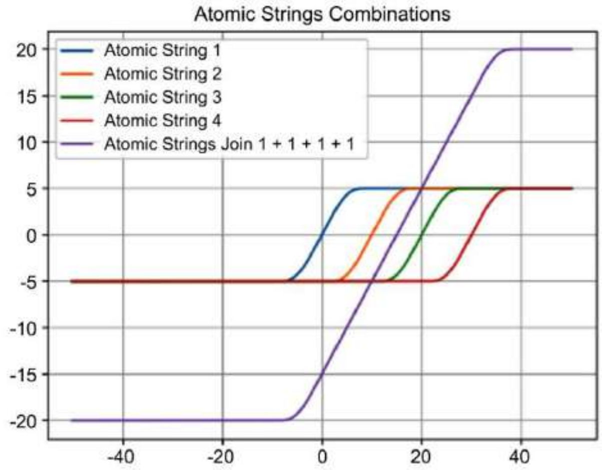

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Fig-4: (a) Atomic String Function (AString) (b) Atomic function as a combination of two AStrings (c) Representation of a straight line segment by summing of AStrings.

The Elementary AString kink function can be generalized in the form

$$AString(x, a, b, c, d = 0) =$$
$$d + c * AString((x - b)/a). \quad (3.10)$$

Importantly, the Atomic Function pulse (3.6) can be presented as a sum of two opposite AString kinks (Fig.4, b) making AStrings and AFs deeply related to each other:

$$up(x, a, b, c) =$$
$$AString\left(x, \frac{a}{2}, b - \frac{a}{2}, c\right) + AString\left(x, \frac{a}{2}, b + \frac{a}{2}, -c\right). \quad (3.11)$$

3.3. Atomic Series and "mathematical atoms"

Atomic and AString Functions (*Atomics*) possess unique approximation properties. Like from "mathematical atoms" [6]-[12], as founders called them, flat and curved smoothed surfaces/functions (Figs.2,3) can be composed of a superposition of Atomics via the so-called Generalized Taylor's Series [7]-[8],[26] (or simply, *Atomic Series*[1]) with an *exact* representation of polynomials of any order

$$\frac{1}{4}\sum_{k=-\infty}^{k=+\infty} kup\left(x - \frac{k}{2}\right) \equiv \sum_{k=-\infty}^{k=+\infty} AString(x - k) \equiv x;$$

$$\sum_{k=-\infty}^{k=+\infty}\left(\frac{k^2}{64} - \frac{1}{36}\right) up\left(x - \frac{k}{4}\right) \equiv x^2,$$

$$x^n \equiv \sum_{k=-\infty}^{k=+\infty} C_k up(x - k2^{-n})$$
$$= \sum_{k=-\infty}^{k=+\infty} C_k (AString(2(x - k2^{-n}) + 1)$$
$$-$$
$$AString(2(x - k2^{-n}) - 1)). \qquad (3.12)$$

Notably, only a limited number of neighboring finite 'atoms' are required to calculate a polynomial value at a given point.
It means Atomics can also represent/atomize any *analytic function* [34] (a function representable by converging Taylor's series) with known calculable coefficients:

$$y(x) = \sum_{m=0}^{\infty} \frac{y^{(m)}(0)}{m!} x^m =$$

m=0∞Bmxm=m=0∞k=−∞k=+∞BmCkupx−k2−m =mk=−∞∞cmkupx−bmkamk=l=−∞l=+∞AStringx ,al,bl,cl. (3.13)

Analytic functions [34] represent a wide range of polynomial, trigonometric, exponential, hyperbolic, and other functions, their sums, derivatives, integrals, reciprocals, multiplications, and superpositions. Therefore, they all can be 'atomized' via superpositions of Atomic and AString Functions with any degree of precision, which is the most important feature.
Instead of sums (3.12), and (3.13), we will be using short notation with localized basis atomic functions $A_k(x)$ and function values y^k at node k assuming summation over repeated indices k:

$$y(x) = A_k\,(x) y^k;$$
$$f(x, y, z) = A_k\,(x, y, z) f^k. \qquad (3.14)$$

3.4. Atomic Solitons

Being solutions of special kinds of nonlinear differential equations with shifted arguments (3.1), (3.8), AStrings and Atomic Functions possess some mathematical properties of lattice solitons [30]-[33] and have been called *Atomic Solitons* [2]-[5]. AString is a solitonic kink whose particle-like properties exhibit themselves in the composition of a line (3.9) and kink-antikink 'atoms' (3.8) (Fig.4). Being a composite object (3.8) made of two AStrings, AF $up(x)$ is not a true soliton but rather a *solitonic atom*, like 'bions' or 'dislocation atoms' [2],[4],[31],[32], as described in [2],[4].

4. ATOMIZATION THEOREMS OVERVIEW

Unique properties of Atomic and AString Functions (Atomics) allow the formulation of *Atomization Theorems*[1],[14]stating how scalar, vector, tensor functions, and solutions of linear and nonlinear differential equations can be represented via Atomic Series - a series like (3.12), (3.13)over Atomic or AString Functions. The proof and detailed description of 13 theorems are provided in [14] and [1].

The fundamental is the following theorem proven in the 1970s [7]-[9] and extended for AStrings in [1],[14].

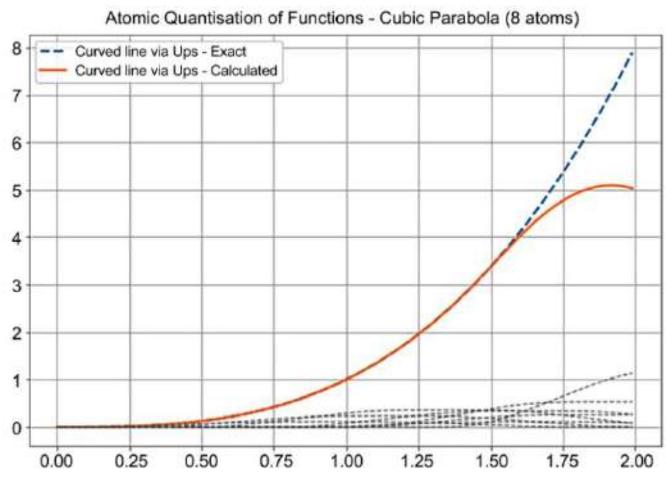
Polynomial atomization theorem. Polynomials of any order can be exactly represented/atomized via the Atomic Series of Atomic and AString Functions:

$$(x^n)^{(n)} = c \sum_{k=-\infty}^{k=+\infty} up(x - k) \equiv c,$$

$$x^n = I_n\left(c \textstyle\sum_{k=-\infty}^{k=+\infty} up(x - k)\right) = \sum_{k=-\infty}^{k=+\infty} C_k up(x - k2^{-n}). \qquad (4.1)$$

$$P_n(x) = x^n + a_1 x^{n-1} + \dots + a_n \equiv \sum_k C_k up\left(\frac{x - ka}{a}\right) =$$
$$\textstyle\sum_k AString(x, a_k, b_k, c_k) = A_k(x) P_n{}^k. \qquad (4.2)$$

Basically, it tells that a polynomial can be represented as a sum of shifts and stretches of atomic pulses, and due to their finiteness only a few neighboring pulses are required to calculate a polynomial value at a given point. This important feature is related to the fundamental property of AF $up(x)$derivatives and integrals (3.3) to be expressed via $up(x)$ itself.Therefore, representing a constant (3.4), (4.1)via the sum of $up(x - k)$and integrating it n-times would yield n −order polynomial finally represented via AFs or AStrings. Moreover, unlike widely used splines of n-order capable only to *exactly* represent n −order polynomial, Atomic splines can compose a polynomial of *any* order, and this is the core feature.



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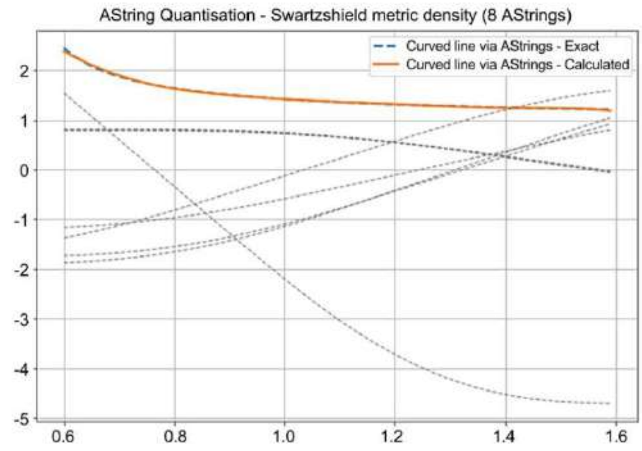

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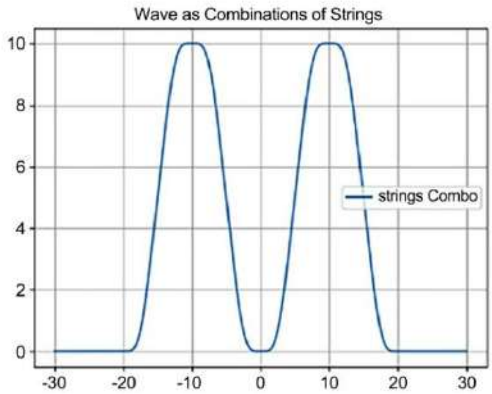

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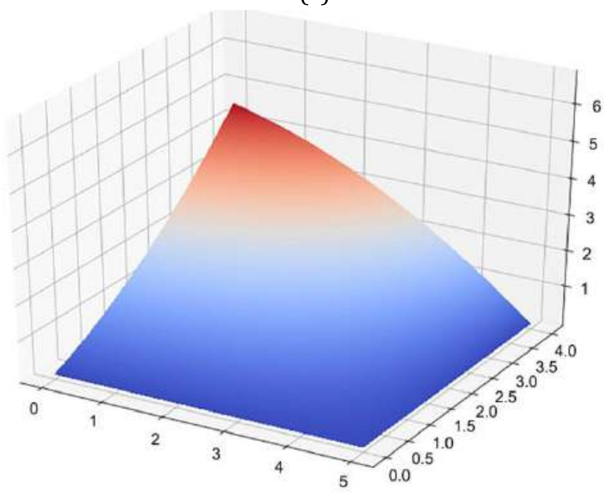

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Fig-5: Representing sections of polynomials and analytic functions with AStrings and Atomic Functions (a) Cubic parabola via 8 Atomic Functions; (b) Schwarzschild metric function; (c) Wave-like formation; (d) 2d surface.

Representing a polynomial via Atomics lead to many important generalizations of Atomization Theorems [1],[14] *for analytic functions*[34] representable via converging Taylor's series by polynomials. It covers a wide range of exponential, trigonometric, hyperbolic, and other analytic functions used in mathematical physics {Fig.5).

Moreover, complex analytic functions [34]$y(x)$thatare sums $y = y_1 + y_2$, products $y = y_1 y_2$, reciprocals $y = 1/y_1$,derivatives $y = y_1'$, integrals $I(y_1)$, and superposition $y = y_1(y_2)$of analytic functions $y_1(x)$, $y_2(x)$would also be analytic functions [34] hence representable by Atomic Series via Atomic and AString Functions with special theorems [1],[14].

The Atomization Theorems can be extended [1],[14] to multi-dimensional analytic functions[1]-[9], [29] and polynomials which are products of one-dimensional polynomials hence representable via multi-dimensional Atomic Functions (3.7):

$$P_{mn} = P_n(x_1, .., x_m) = \prod_{i=1}^{m} P_n(x_i) =$$
$$= \sum_k \prod_{i=1}^{m} up_i(x_i, a_{ik}, b_{ik}, c_{ik}) =$$
$$\sum_k \boldsymbol{UP}(\boldsymbol{a_k}, \boldsymbol{b_k}, \boldsymbol{c_k}) = \sum_l \boldsymbol{AString}(\boldsymbol{a_l}, \boldsymbol{b_l}, \boldsymbol{c_l}) =$$
$$A_k(x_1, .., x_m) P_{mn}{}^{k}. \quad (4.3)$$

In general, polynomic, trigonometric, exponential, and other analytic functions are the solutions of some linear differential equations (LDE) implying that Atomization Theorems can be extended to differential equations [7]-[12], [26], with the theory described in [1],[14].

5. ATOMIZATION THEOREMS IN GENERAL RELATIVITY

The Atomization Theorems [1],[14] can be extended to nonlinear differential equations of General Relativity (GR) [15]-[22]as one of the most fundamental theories of modern physics.

5.1 Atomization Theorems for metric, curvature, and Ricci tensors

Considering together multidimensional Atomic Series and Atomization Theorems leads to the following theorems [1],[14] important for GR.

Tensor's atomization theorem. First $\partial_i = \frac{\partial}{\partial x_i}$ and second derivatives $\partial_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}$ as well as the metric tensor g_{ij} defining interval on a curved surface $ds^2 = g_{ij}(x_n) dx_i dx_j$ preserve analyticity and being applied to analytic functions $y_k(x_l)$ lead to analytic functions representable by Atomic Series over Atomic and AString Functions.

Proof of the theorem provided in [1],[14]is based on the observation that both first and second-derivative operators *preserve analyticity* because derivatives of multidimensional polynomials $B_{lm} {x_l}^m$ would also be polynomials exactly representable via multidimensional Atomic Functions and AStrings (3.7), (4.3) using Atomic Series (3.13). For curved spacetime surfaces/geometries described by some analytic functions $\tilde{x}_i = \tilde{x}_i(x_j); d\tilde{x}_i = \frac{\partial \tilde{x}_i}{\partial x_j} dx_j$, the derivatives and their multiplications would be analytic, hence also representable by Atomics.

This theorem can be understood in another way by noting that all derivatives and integrals of Atomics are expressed via themselves (3.3), (3.8), and if space geometry analytic functions $\tilde{x}_i(x_j)$ are the sum of Atomics, then all derivatives and metric tensors would also be some Atomics combinations:

$$g_{ij}(x_n) = \sum_{ijnk} up(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} AString(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}). \quad (5.1)$$

This theorem means that for analytic spacetime geometries/configurations, their deformations, curvatures, metrics, and geodesics would also be some Atomics superpositions, with a range of analytical surfaces and spacetime metrics known in GR [15]-[17] described later. Furthermore, due to the properties of analytic function superpositions to preserve analyticity, the last theorem can be extended [1],[14] to nonlinear Ricci tensors important in GR [15]-[17].

Ricci tensor atomization theorem. Nonlinear Ricci tensor R_{jk} and Christoffel operators Γ_{ij}^k preserve analyticity and applied to analytic functions would yield analytic functions representable/atomizable by Atomic Series via Atomic and AString Functions.

Proof[1],[14] is based on the observation that Christoffel operators [16], [17], which include multiplications of functions to their spatial derivatives, transform analytic metric tensor functions (5.1) representable by polynomials into more complex polynomials representable by Atomics via Atomic Series (4.2). Similarly, Ricci tensors are also a combination of derivatives and multiplications of Christoffel symbols [16], [17] which preserve analyticity, hence representable via Atomics:

$$\Gamma_{ij}^k = \frac{1}{2} g^{kl}(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij});$$

$$R_{jk} = \partial_i \Gamma_{jk}^i - \partial_j \Gamma_{ik}^i + \Gamma_{ip}^i \Gamma_{jk}^p - \Gamma_{jp}^i \Gamma_{ik}^p \quad (5.2)$$

$$R_{ij}(x_n) = \sum_{ijnk} up(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} AString(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}). \quad (5.3)$$

This theorem can be intuitively understood in the sense that polynomials are 'hard to destroy' even by complex nonlinear differential operators because their multiplications, derivatives, integrals, and some superpositions would also be polynomials representable by Atomics. It also means that not only spacetime metrics but also curvature tensors can be 'atomized' using shifts and stretches of finite AString and Atomic Functions.

5.2. Atomization Theorem for General Relativity

The sequence of Atomization Theorems [1],[34]finally converges into the following theorem for Einstein's General Relativity [16]-[22] proven in [1],[14].

Atomic Spacetime Theorem. For analytic manifolds, Einstein's curvature tensor preserves analyticity and yields spacetime shapes, deformations, curvatures, and matter/energy tensors representable via multi-dimensional Atomic and AString Functions superpositions. Solutions of General Relativity equations can be represented/atomized by converging Atomic Series over finite Atomic and AString Functions:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = \sum_{\mu\nu i} \boldsymbol{UP}(x_i, \boldsymbol{a_{\mu\nu i}}, \boldsymbol{b_{\mu\nu i}}, \boldsymbol{c_{\mu\nu i}}) = \sum_{\mu\nu i} \boldsymbol{AString}(x_i, \boldsymbol{a_{\mu\nu i}}, \boldsymbol{b_{\mu\nu i}}, \boldsymbol{c_{\mu\nu i}}). \quad (5.4)$$

Proof provided in [1],[14] is based on the observation that for analytic manifolds - spacetime geometries described by analytic functions $\tilde{x}_i = \tilde{x}_i(x_j)$representable by converging Taylor's series - the metric tensors $g_{\mu\nu}$ composed of derivatives and their multiplications would also be some analytic functions. Being injected into Christoffel operators (5.2) and then Ricci tensors $R_{\mu\nu}$, they would yield another set of analytic functions representable by Taylor's series because the derivatives, multiplications, and superposition of analytic functions would also be analytic. The curvature scalar $R = g^{\mu\nu} R_{\mu\nu}$ in (5.4) preserves analyticity because of the cross-multiplication of polynomials and their derivatives would also be polynomials. Injected into (5.4), those tensors produce Einstein's tensor $G_{\mu\nu}$ and energy-momentum tensor $T_{\mu\nu}$ ($\frac{8\pi G}{c^4}$ is a constant) supposedly representable by polynomials via multi-dimensional Taylor's series. Because a polynomial of any order is exactly representable/atomizable via series over Atomic and AString Functions (4.2), the spacetime curvature, metric, and energy/momentum tensors would be the superpositions of multi-dimensional Atomic $\boldsymbol{UP}$ and $\boldsymbol{AString}$ functions (4.3), derivatives of which are expressed via themselves. Due to fundamental relation (3.8) $up(x) = AString'(x) = AString(2x+1) - AString(2x-1)$, the Atomic Function $up(x)$is a sum of two AStrings which can within one model, compose straight $x = \sum_k AString(x, a, ka, a)$ and curved $\tilde{x} = \sum_k AString(x, a_k, b_k, c_k)$ lines from elementary AString pieces resembling quanta (§2,6).

In a nutshell, this theorem tells that the spacetime field is representable via series over AStrings and Atomic Functions, the derivatives of which are expressed via themselves meaning the spacetime shape, deformations, curvatures, and energy/momentum tensors can be represented as some superposition of finite Atomics. It offers an 'atomic model' of spacetime [1]-[5] where 'atoms' are associated with 'mathematical atoms' from the theory of Atomic Functions.

Let's note that *atomization* is not a simple *discretization* of space – separation of a volume into adjacent finite elements [23],[24],[39]. Here, the 'finite elements' (AStrings) are overlapping (§2,Figs.1,5) and capable to describe both expansions of space (3.9) and localized solitonic atoms $up(x)$(3.8). More precisely, atomization includes space discretization on finite elements on some lattice but demands smooth connections between adjacent finite elements (§2) yielding smooth Atomic Functions.

Let's note that reversing Atomization Theorems allows deriving Atomic and AString Functions from General Relativity equations (5.4), as described in [14].

6. ATOMIC SPACETIME MODEL

6.1Spacetime Atomization model

Atomization Theorems provide a theoretical foundation for atomization/quantization of spacetime field based on Atomic and AString Functions when GR equations and solutions, along with Ricci, curvature, and metric tensors, can be represented via Atomic Series over multidimensional Atomic and AString Functions (3.7):

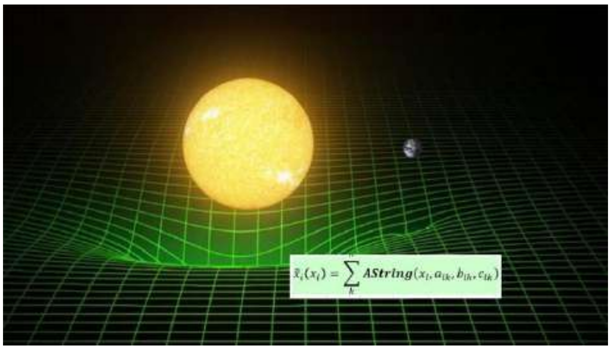
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = \sum_{\mu\nu i} \boldsymbol{UP}(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}) = \sum_{\mu\nu i} \boldsymbol{AString}(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}), \quad (6.1)$$

$$R_{ij}(x_n) = \sum_{ijnk} \boldsymbol{UP}(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} \boldsymbol{AString}(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}), \quad (6.2)$$

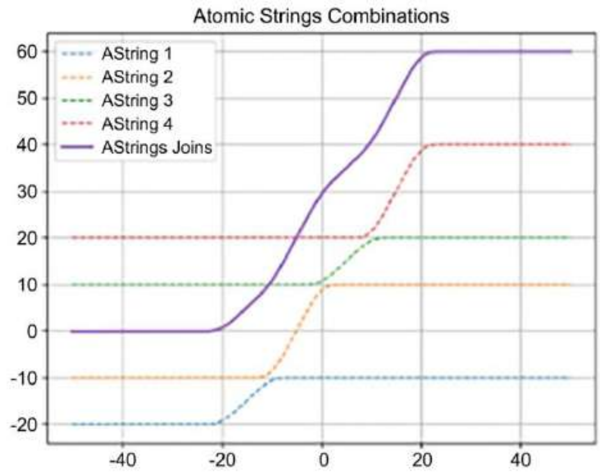
$$g_{ij}(x_n) = \sum_{ijnk} \boldsymbol{UP}(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} \boldsymbol{AString}(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}), \quad (6.3)$$

$$\tilde{x}_i(x_l) = \sum_k \boldsymbol{AString}(x_l, a_{lk}, b_{lk}, c_{lk}). \quad (6.4)$$

These formulae express the mathematical fact that it is possible to compose analytical manifolds (Figs.3-6) by adjusting parameters of localized Atomic and AString Functions to reproduce different metrics, or like in the Legogame, compose a smooth shape from 'elementary pieces' smoothly connected to each other resembling ideas of spacetime quantization. If finite Atomics, for which derivatives are expressed via themselves, represent spacetime shape $\tilde{x}_i(x_l)$(6.4), the series over Atomics would also describe spacetime deformations, curvatures, metrics, Ricci's, Einstein's, and energy-momentum tensors.



(a)



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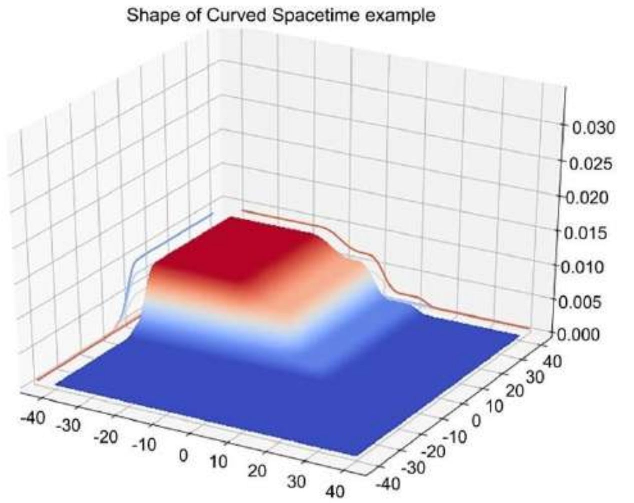


(c)

Fig-6: (a) Curved spacetime composed of AStrings (b) Joining AStrings of different heights simulates spacetime curving (c) Curved spacetime geodesics represented via joints of 3D solitonic atoms.

The presentation of spacetime fields via the Atomic Series is conceptually similar to a presentation of complex functions and fields via the Taylor or Fourier series over trigonometric functions for which derivatives, integrals, and superpositions are also expressed via themselves. But here, the series is based on *finite* Atomic Functions leading to ideas of quantization. Because AString can compose a line and a curve from 'elementary pieces' resembling quanta (§2,6,7) one can envisage a spacetime field as a complex lattice network of flexible spacetime quanta (Figs.4,6) which can stretch and overlap to reproduce different smooth shapes. The notion of 'quantum' here is not directly related to Quantum Mechanics and Quantum Gravity [20],[21],[40] but rather to the finiteness and hypothetical indivisibility of 'solitonic atoms' capable to compose shapes and fields via superpositions.

6.2. Atomization of known General Relativity solutions

The idea of Atomic Spacetime atomization can be demonstrated for known GR solutions [1]-[5],[15]-[22].

Einstein-Minkowski solution$T_{\mu\nu} = 0, g_{ij} = 1$ for homogeneous uniform spacetime/universe [15]-[17] is simply atomizable/quantizable via translations of identical overlapping AString quanta (§2, Fig.3) [1]-[5] in vector notation:

$$\boldsymbol{AQuantum}(x_1, x_2, x_3, t, a, \rho, c_l) = AString(x_1, a, a, \rho a)\boldsymbol{e_1} + AString(x_2, a, a, \rho a)\boldsymbol{e_2} + AString(x_3, a, a, \rho a)\boldsymbol{e_3} + AString(t, a/c_l, a/c_l, \rho a/c_l)\boldsymbol{e_t}, \quad (6.5)$$

or schematically in vector form (Figs. 3, 4, 6, 7c)

$$\boldsymbol{UniformSpace}(x_1, x_2, x_3, t) = \sum_k \boldsymbol{AQuantum}(x_1, x_2, x_3, t, a, \rho, c_l). \quad (6.6)$$

Friedmann's solution for expanding a spatially homogeneous universe with metric [15]-[17]

$$ds^2 = a(t)^2 d\bar{s}^2 - c^2 dt^2; \quad d\bar{s}^2 = dr^2 + S_k(r)^2 d\Omega^2 \quad (6.7)$$

includes analytic function $S_k(r)$ representable via Atomic Series (3.13):

$$S_k(r) = r\operatorname{sinc}(r\sqrt{k}) = r - \frac{kr^3}{6} + \frac{kr^5}{120} - \cdots = \sum_k c_k up\left(\frac{r - b_k}{a}\right) =$$

$$= \sum_k AString(r, a_k, b_k, c_k). \quad (6.8)$$

Scale factor $a(t)$ [16]-[18] being an analytic power function [34] is also representable via Atomics:

$$a(t) = a_0 t^{\frac{2}{3(w+1)}}; a(t) \sim t^{\frac{2}{3}}, w = 0;$$

$$a(t) \sim t^{1/2}, w = 1/3, \quad (6.9)$$

$$a(t) = \sum_k up(t, a_k, b_k, c_k) = \sum_l AString(t, a_l, b_l, c_l). \quad (6.10)$$

Schwarzschild solution (Fig.7) for radial bodies and black holes has spacetime metric [15]-[19],[22]

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2 d\Omega;$$

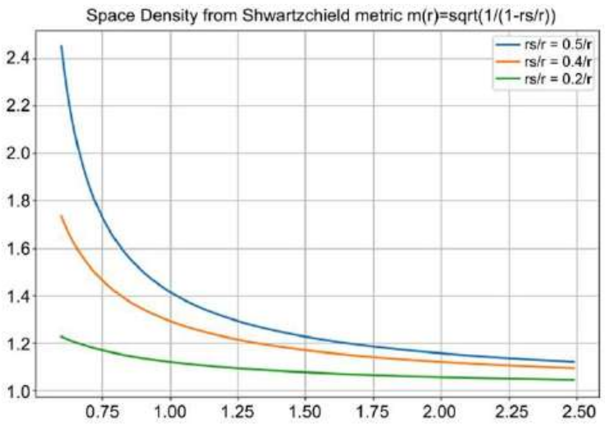
$$A(r) = \left(1 - \frac{r_s}{r}\right); \ B(r) = \left(1 - \frac{r_s}{r}\right)^{-1}. \quad (6.11)$$

Analytic (outside of singularity) function $A(r)$ and its reciprocal $B(r)$ (also analytic [34]) representable via converging Taylor's series is also representable via Atomics (4.4):

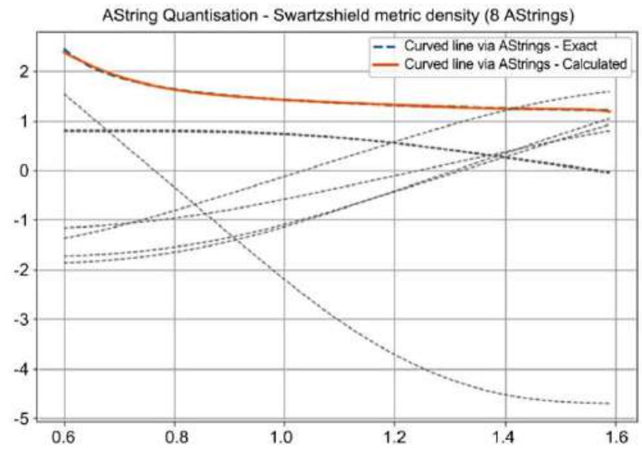
$$A(r) = \sum_k c_k up\left(\frac{r - b_k}{a}\right) =$$

$$\sum_k AString(r, a_k, b_k, c_k), r \neq 0. \quad (6.12)$$

In summary, the atomization of known GR solutions confirms the main idea that analytic spacetime fields are representable via the superposition of finite AStrings and Atomic Functions.



(a)



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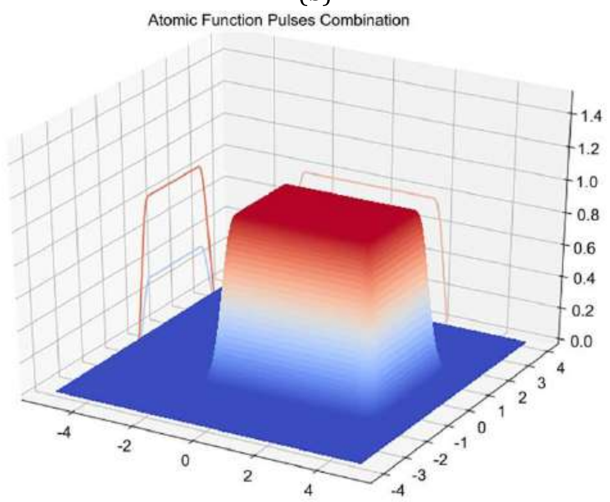

(c)

Fig-7:(a) Space density function from Schwarzschild GR solution (b) Representing Schwarzschild metric via AStrings (c) Uniform spacetime field as a superposition of solitonic atoms.

7. ATOMIC SPACETIME PROPERTIES

Representing the spacetime field as a superposition of Atomic and AString Functions offers the following interpretations under research now and discussed in detail in [1]-[5],[14].

1. No point in space – space is rather a superposition of finite 'solitonic atoms' $cup((x-b)/a)$.
2. Spacetime is a field composed of superpositions of 'solitonic atoms' $cup((x-b)/a)$ on a lattice with size a which may be associated with Planck's length $a \sim 1.6 \times 10^{-35}$ m (Figs. 6,7).
3. Finite 'solitonic atoms' may be associated with some flexible quanta (§2,6)$cup((x-b)/a)$with intensity c, width a, and location b.The flexibility of quanta is important to uphold Einstein's Special Relativity [15]-[22] stating that lengths may stretch and shrink.
4. Atoms/quanta interact in overlapping zones with the preservation of smoothness (§2, §3.1).
5. Spacetime 'emerges' between quanta/atoms $x \equiv AString\left(x-\frac{1}{2}\right) + AString\left(x+\frac{1}{2}\right)$(§2,3).
6. Spacetime atomization is not a simple space discretization, due to overlapping zones.
7. Spacetime is a flexible 'fabric' of solitonic atoms adjusting their locations and intensities to reproduce variable fields between atoms (§6.1, §6.2).
8. Expansion of spacetime means bigger quanta (6.6) with growing width (§6.2).
9. Space 'solitonic atoms' are mathematically 'made of' AStrings $up(x) = AString(2x+1) + AString(1-2x)$(3.8).
10. AString kinks pointing in one direction model spacetime expansion $x = \sum_k AString(x-k)$while opposite AStrings compose finite atom $up(x) = AString(2x+1) + AString(1-2x)$(§2, 3).
11. AString models a 'quantum of length' capable to represent a length function $L(x)$ in uneven space with density $\rho(x)$ as a sum of AStrings (§2, Figs.1,2):

$L(x) = \int_0^x d\tilde{x}(x) =$

$\int_0^x \rho(x)dx = \int_0^x \sum_k up(x, a_k, b_k, c_k)\, dx =$

kAStringx,ak, bk, ck

12. AString may model a 'metriant' – space quantum in some General Thermodynamics theories [35]which led to the notion of AString metriantin [1]-[5].
13. Spacetime has energy as a sum of discrete energies ca^3(3.7) of multiple solitonic atoms (§6.1).
14. Spacetime may be solitonic in nature [1]-[5] and composed of overlapping Atomic Solitons (§6).
15. Spacetime field is both discrete and continuous; discrete, being composed of finite solitonic atoms, and continuous, being united with smooth connections

between 'atoms' (§2, §6.1).12. It looks like smooth and finite AString kinks are 'blended' into smooth curves.

16. Spacetime is fractalic due to fractalic properties of a 'length' composed of 'pieces' [2]-[5].

17. Atomic spacetime model introduces new constants, mainly lattice size a which may be associated with Planck's length $a \sim 1.6 \times 10^{-35}$ m, or another fractalic lattice size for macroscopic fields.

18. Atomic Spacetime is a microscopic model which cannot explain the macroscopic fields' relationships (for example, why spacetime is curving around big masses) butcan provide microscopic fields interpretations as a superposition of overlapping solitonic atoms stretching their locations to reproduce variable fields.

8. CONCLUSIONS AND RESEARCH DIRECTIONS

8.1. Atomic Spacetime and fields future research directions

Theory of Atomic Functions [7]-[14] evolving since the 1970s towards AString Functions, Atomic Solitons [2]-[5], and Atomization Theorems [1], [14] appearing in 2017-2022 provides a theoretical foundation for applications of AStrings and Atomic Functions in many physical theories being researched further [1]-[5],[14] including spacetime physics. The common feature of these theories is the unified description of spacetime and other fields as superpositions of flexible overlapping solitonic atoms $cup((x-b)/a)$ made of two AStrings $up(x) = AString(2x+1) - AString(2x-1) = AString'(x)$. It means that mathematically field distributions are just complex combinations/lattices/networks of flexible AStrings.

Extensions of AString pointing in one direction describe spacetime expansion while opposite AStrings form finite 'solitonic atoms' resembling localized matter. Moreover, due to properties (3.1) and (3.8) of Atomics to have derivatives expressed via the functions themselves, not only the fields but their derivatives expressing fields deformations and curvatures, would also be some AStrings combinations. This invites the hypothesis raised in [2]-[5] whether AString mathematically describes some fundamental solitonic object from which *everything* is made, and having a common mathematical 'ancestor', different fields may be deeply related to each other.

The obvious candidate for a 'particle of everything' is a new kind of string from string theory [41]. Rather than consider a string as a 'linearly vibrating' filament, one can hypothesize that a string with length a may vibrate 'nonlinearly' with compact $cup(x/a)$ 'solitonic atom' shape composed of two AStrings (Fig.4) with intensity/amplitude c and energy/integral ca (ca^3 in 3D). Two neighboring strings may overlap and produce either constant or variable smooth field while continuous space x 'emerges' between strings $x \equiv aAString\left(x - \frac{a}{2}\right) + aAString\left(x + \frac{a}{2}\right), x \in \left[-\frac{a}{2}, \frac{a}{2}\right]$ and extends by adding more strings $x \equiv \sum_k aAString((x-ka)/a)$(§2,6). Then, electron, quark, or Higgs 'particles' may be the spatial excitation of strings with their own intensity, energy, and size [1],[14]. Because these fields have a common 'string ancestor', they become deeply related to each other, with the preservation of energy during exchanges. The most important in this 'string' model is that strings can overlap with preservation of smoothness in between which leads to AString and Atomic Functions as per §2. But also, strings can stretch hence break the uniformity of spacetime and produce variable fields described by more complex Atomic Series (3.13).

Another idea related to AStrings is that the Atomic String model assumes the existence of a 'string with length' a embedding the concepts of a 'length', 'dimensionality', and 'lattice'. It looks like every particle (electron, boson) apart from unique physical characteristics (eq electric charge) includes some 'quanta of space', or metriants, as A. Veinik [35], call them in the 1980s. Like the Higgs field giving matter 'a property of mass' [45], metriants 'give the matter the property of size' and 'order of location' [35]. AString function not only offers a mathematical model for the metriant (§2,6) but also allows the building of 'solitonic atoms' $up(x) = AString(2x+1) - AString(2x-1)$ capable to compose different fields in superposition. This concept of AString metriants as 'common blocks' of fields has synonyms like 'quantum of length', 'elementary distortion of spacetime', and 'ripple/excitation' of spacetime used by other authors [18],[35],[39],[44].

Hopefully, verified by physicists and string theorists, these purely mathematical string and metriant models [1]-[5] rooted in 50 years of history of Atomic Functions [7]-[14] may contribute to spacetime physics, quantum field theories, and unified theories of nature [18],[22],[35],[37],[43]-[45].

8.2. Atomic Spacetime and "Atomic Theory" of A. Einstein

To conclude the paper, it is tempting to recall A. Einstein's 1933 lecture "On the method of theoretical physics" [15] cited below with some highlights where he envisaged the prospects of a novel *"atomic theory"* based on *"mathematically simplest concepts and the link between them"* to solve some *"stumbling blocks"* of continuous field theories to describe quantized fields.

"The important point for us to observe is that all these constructions and the laws connecting them can be arrived at by the principle of looking for the mathematically simplest concepts and the link between them. In the limited number of the mathematically existent simple field types, and the simple equations possible between them, lies the theorist's hope of grasping the real in all its depth. Meanwhile the great stumbling-block for a field-theory of this kind lies in the conception of the atomic structure of matter and energy. For the theory is fundamentally non-atomic in so far as it operates exclusively with continuous functions of space, in contrast to classical mechanics, whose most important element, the material point, in itself does justice to the atomic structure of matter. The modern quantum theory in the form associated with the names of de Broglie, Schrodinger, and Dirac, which operates with continuous functions, has overcome these difficulties by a bold piece of interpretation which was first given a clear form by Max Born. According to this, the spatial functions which appear in the equations make no claim to be a mathematical model of the atomic structure. Those functions are only supposed to determine the mathematical probabilities to find such structures, if measurements are taken, at a particular spot or in a certain state of motion This notion is logically unobjectionable and has important successes to its credit. Unfortunately, however, it compels one to use a continuum the number of whose dimensions is not that ascribed to space by physics hitherto (four) but rises indefinitely with the number of the particles constituting the system under consideration. I cannot but confess that I attach only a transitory importance to this interpretation. I still believe in the possibility of a model of reality - that is to say, of a theory which represents things themselves and not merely the probability of their occurrence. On the other hand, it seems to me certain that we must give up the idea of a complete localization of the particles in a theoretical model. This seems to me to be the permanent upshot of Heisenberg's principle of uncertainty. But an atomic theory in the true sense of the word (not merely on the basis of an interpretation) without localization of particles in a mathematical model is perfectly thinkable. For instance, to account for the atomic character of electricity, the field equations need only lead to the following conclusions: A region of three-dimensional space at whose boundary electrical density vanishes everywhere always contains a total electrical charge whose size is represented by a whole number. In a continuum-theory atomic characteristics would be satisfactorily expressed by integral laws without localization of the entities which constitute the atomic structure. Not until the atomic structure has been successfully represented in such a manner would I consider the quantum-riddle solved."

Interestingly, some of Einstein's aspirations of a novel "atomic theory" with "simplest concepts and links between them" based on finite "regions of space" with "atomic structure" can be realized with the described Atomic Spacetime theory based on Atomic and AStrings Functions evolving since the 1970s.Indeed, the theory is "atomic" assuming atomizing/quantizing spacetime field with finite Atomic String Functions. Atomic Functions, as 'solitonic atoms' made of two AStrings (3.8), are "*simple concepts*". "*Links between them*", in the form of overlapping superpositions (3.9), (3.12), (6.1), allow describing flat and curved spacetime and other physical fields. The theory also overcomes "*stumbling blocks*" of theories dealing "*…exclusively with continuous functions of space"* [15]; here, atomized spacetime is both discrete and continuous (§6, 7). Also, Einstein's *"…region of three-dimensional space at whose boundary electrical density vanishes everywhere*" [15] naturally leads to a *finite* Atomic Function (Fig.2) with discrete energy (integral (3.7)) levels ca^3. Atomic Spacetime theory seems to support another Einstein's quote: "*I have deep faith that the principle of the universe will be beautiful and simple"*. This "principle" may be realized with the simple model of spacetime and field composition from Atomic Solitons composed of simple AString metriant functions.

Acknowledgement

The atomic spacetime theory expands the theory of Atomic Functions pioneered in the 1970s by the author's teacher Academician of the National Academy of Sciences of Ukraine Vladimir Logvinovich Rvachev, and this paper is dedicated to his genius.

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