International Journal of Innovative Research in Sciences and Engineering Studies (IJIRSES)

www.ijirses.com *ISSN: 2583-1658 | Volume: 2 Issue: 12 | 2022*

On Derivation of Non-Parametric Weighted Linear Models

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Abstract: *Rank Based approaches outperform least squares procedures when the data deviates from normality and/or contains outliers. Weights can be added to these approaches to create weighted strategies (WT). We demonstrate how to construct WT-estimates using Rank-based regression in this paper. Rank-based estimators were developed to provide a nonparametric, robust alternative to traditional likelihood or least squares estimators. Jurecková and Jaeckel (1971) were the first to use rank-based regression (1972). More importantly, we offer a collection of Weights that may be used to do a robust analysis of a linear model based on WT-estimates. To develop the set of functions, we chose to use the R statistical software packages. Because R is free and runs on a multitude of platforms, WT estimation and inference are now available to everyone.*

Keywords: *estimation, inference, linear models, R functions, rank-based procedures, robust, weighted estimates.*

Introduction

The work focused on employing the reciprocal of the sample inclusion probability as weights, using various modified and unmodified weights, to generate Non Parametric estimators with higher relative precision and low bias.

1. The Model

Suppose the data is produced according to a function:

$$f(y \mid x; \theta)\, g(x) \qquad (1.1)$$

where y is a response variable which is multivariate and x is a continuous or discrete vector of covariate variables and

$$f(y \mid x; \theta) \qquad (1.2)$$

is the regression part of the function. The marginal distribution of x is denoted by g(x) which for this study we have used Gaussian density to represent, as shown below

$$k(u) = \frac{1}{\sqrt{(2\pi)}} e^{\left(\frac{-u^2}{2}\right)} \qquad (1.3)$$

where

$$u = \frac{x_1 - mean(x_1)}{s\tan.dev(x_1)}$$

We describe the conditional distribution of y given x_1 as θ. The likelihood is given by

$$\prod f(y|x;\theta) \qquad (1.4)$$

as explained by Kamun et. al. (July 28, 2021, August 31, 2021).

2. Rank-Based Estimation

For a regression, use paired data (Y_i, X_i).

$$Y_i = \mu + \beta X_i + \varepsilon_i,\ 1 \le i \le n \qquad (2.1)$$

The errors ε_i in (2.1) are supposed to be independent and uniformly distributed, however they are not always normal and may be heavy-tailed. Assume for the sake of simplicity that is one dimensional.

Then (2.1) is a straightforward linear regression. However, much of the following can be easily extended to higher dimensions, in which case (2.1) is a multiple regression.

Provided β, specify $R_i(\beta)$ as the rank (or mid-rank) of $Y_i - \beta X_i$as one of { $Y_j - \beta X_j$ }. Therefore $1 \le R_i(\beta) \le n$. The rank-regression estimator $\hat{\beta}$ would be any number of β that minimizes the sum

$$D(\beta) = \sum_{i=1}^{n} R_i^c(\beta)(Y_i - \beta X_i) \qquad (2.2)$$

Such that

$$R_i^c(\beta) = R_i(\beta) - (n+1)/2 \qquad (2.3)$$

are the centered ranks or mid-ranks.

As of

$$\sum_{i=1}^{n} R_i^c(\beta) = 0$$

in (2.3), we can subtract a constant from $Y_i - \beta X_i$ in (2.2) without influencing D (β). That is,

$$D(\beta,\mu) = \sum_{i=1}^{n} R_i^c(\beta)(Y_i - \beta X_i - \mu)$$

$$= \sum_{i=1}^{n} R_i^c(\beta)(Y_i - \beta X_i) = D(\beta) \qquad (2.4)$$

for all μ. Since

$$D(\beta)=\sum_{i=1}^{n} R_i^c(\beta)(Y_i-\beta X_i-\mu) \text{ and}$$

$$\bar{\mu}=\bar{Y}-\beta\bar{X}$$

and

$$\sum_{i=1}^{n}(Y_i-\beta X_i-\bar{\mu})=0$$

and since $Y_i - \beta X_i < Y_j - \beta X_j$ implies $R_i^c(\beta)<R_j^c(\beta)$ as a result of D(β) > 0 for all β unless $Y_i - \beta X_i$ is constant.

As previously stated,, the rank regression slope estimator for β in (2.1)is an example of

$$\min_{\beta} D(\beta)=D(\hat{\beta}) \quad (2.5)$$

More specifically, both D(β) in (2.2) and $\hat{\beta}$ in (2.5) are functions of the residuals $Y_i - \mu - \beta X_i$ in (2.4) and (2.1).

The classical least-squares estimators of μ and β are obtained by minimizing

$$C(\beta,\mu)=\sum_{i=1}^{n}(Y_i-\mu-\beta X_i)^2 \quad (2.6)$$

in place of (2.5). The least-squares estimator $\hat{\beta}_c$ from (2.6) is

$$\hat{\beta}_c=\frac{\sum_{i=1}^{n}(Y_i-\bar{Y})(X_i-\bar{X})}{\sum_{i=1}^{n}(X_i-\bar{X})^2} \quad (2.7)$$

3. An Algorithm for Finding $\hat{\beta}$ in (2.5) that is Nearly as Simple is given Below.

First, take note of the function D(β) in (2.2) is a linear function of β except at values of β for which the ranks R_i^c change. These values pertain to pairs of integers (*i*, *j*) (*i*≠*j*) for which $Y_j - \beta X_j = Y_i - \beta X_i$, or equivalently (if $X_i \neq X_j$) if $\beta = (Y_j - Y_i)/(X_j - X_i)$ for some *i* and j. Let W_k be the sorted difference quotients

$$\{W_k:1\le k\le N\}=\{(Y_j-Y_i)/(X_j-X_i):1\le i<j\le n, X_i\ne X_j\} \quad (3.1)$$

To ensure completeness, set $W_0 = -\infty$ and $W_{N+1} = \infty$. Then D(β) is linear in each interval (W_k, W_{k+1}) $(0 \le k \le N)$. Since the mid-ranks R_i and R_j are the same if $Y_i - \beta X_i = Y_j - \beta X_j$, it follows that D(β) is continuous at each $\beta = W_k$, and hence is continuous (and piecewise linear) for all β.

Now consider a point of discontinuity $\beta = W_k$ in the slope of D(β). Then there exist integers *i*, $_j$ such that for sufficiently small ε> 0

$$\begin{aligned} Y_i-(W_k-\varepsilon)X_i &< Y_j-(W_k-\varepsilon)X_j \\ Y_i-W_kX_i &= Y_j-W_kX_j \\ Y_i-(W_k+\varepsilon)X_i &> Y_j-(W_k+\varepsilon)X_j \end{aligned} \quad (3.2)$$

That is, $Y_i - \beta X_i$ crosses $Y_j - \beta X_j$ from below at $\beta = W_k$. This implies that $X_i<X_j$, and also that

$R_i(Y - \beta X) < R_j(Y - \beta X)$ at $\beta = W_k - \varepsilon$. Thus R_i raises by one and R_j lowers by one as β crosses through $\beta = W_k$ from below. This means that the slope of D(β) increases by $-X_i - (-X_j) = X_j - X_i > 0$.

Thus the slope of D(β) always raises as β crosses through $\beta = W_k$ from below, and the slope of D(β) is raising for $-\infty < \beta < \infty$. Therefore D(β) is convex in addition to being piecewise linear and continuous. Since D(β) is convex, continuous, and piecewise linear, D(β) reaches its minimum either at a unique node

$\beta = W_k$ or else on a unique interval (W_{k-1}, W_k).

By the definition (3.1), the differences $Y_i - \beta X_i$ have the same relative order for $\beta < W_1$, which is the same relative order for $\beta \to -\infty$, which is the same order as the X_i. Similarly, $Y_i - \beta X_i$ have the opposite order of X_i if $\beta > W_N$. Thus

$$R_i(Y-\beta X)=\begin{cases} =R_i(X), \beta<W_1 \\ =n+1-R_i(X), \beta>W_N \end{cases}$$

In particular by (3.2)

$$Slope(D(\beta))=\begin{cases} =-\sum_{i=1}^{n}R_i^c(X)(X_i), \beta<W_1 \\ =\sum_{i=1}^{n}R_i^c(X)(X_i), \beta>W_N \end{cases}$$

Since $\sum_{i=1}^{n}R_i^c(X)\bar{X}=0$, it follows that

$$Q=\sum_{i=1}^{n}R_i^c(X)X_i>0 \quad (3.3)$$

unless the X_i are constant.

Theorem 3.1. Let (i_k, j_k) be the integers (*i*, *j*) corresponding to k in the definition of $W_k$in (3.1). Define

$S_0 = -Q$ for Q in (3.3) and

$$S_k = -Q + \sum_{p=1}^{k} \left| X_{jp} - X_{ip} \right| \quad (3.4)$$
$$k_0 = \min\{k : S_k > 0\}$$

for 1 ≤ k ≤ N. Then S_k is the slope of D(β) for $W_k < \beta < W_{k+1}$. The rank -regression estimator $\hat{\beta}$ defined by the minimum of D(β) in (1.5) is

$$\hat{\beta} = \begin{cases} = W_{k_0} = \dfrac{Y_{jk_0} - Y_{ik_0}}{X_{jk_0} - X_{ik_0}}, S_{k_0-1} < 0 < S_{k_0} & (2.5a) \\ = \dfrac{W_{k_0-1} + W_{k_0}}{2}, S_{k_0-1} = 0 < S_{k_0} & (2.5b) \end{cases}$$

Remarks:

(1) Theorem 2.1 provides a simple algorithm for estimating $\hat{\beta}$.

(2) Since $\hat{\beta} = W_{k0}$ where k_0 depends on S_k, the estimator $\hat{\beta}$ can be viewed as a "weighted median" of the difference quotients $W_k = (Y_j - Y_i)/(X_j - X_i)$ (Hollander and Wolfe, 1999).

4. Weighted Non Parametric Weights

In our study, we propose two novel weighted system of regression estimating equation for estimating coefficients for small sample studies: the small sample weighted conditional pseudo-likelihood estimator (WCPE) model and the small sample semi-parametric weighted likelihood estimator (SPW) model. By fitting a regression model to the establish sample weights against the sample variables and/or response variable, which is the requirement for the two estimates to be obtained we end up estimating the conditional expectation of the weights.

We modified the original sample weights, for WCPE, where the estimated conditional expectation of the weights is used. For our study, both response variables and predictor variables were used to find estimated weights. The SPW and WCPE models are different from the design-based weighted estimators using suitable modification on the original sample weights.

Consider a finite population Ω of individuals. Let y denote the response variables and v(x′ z′) ′, denote the vector of all measured covariates, where x are covariates associated with the outcome and z are the sample variables used in the process of sample selection. x and z may or may not have common variables.

Let n be the size of the observed sample S. The probability that individual, *i*, *i* = 1, 2, 3, ..., n is included in the sample is denoted by π_i. The base sample weight w_i is defined as a reciprocal of the sample inclusion probability π_i so $\pi_i = 1/w_i$. We refer to the final sample weight as the sample weight of individual *i* and denoted by w_i.

We have assumed in this study that the observed data are the data for the simulated units, for individuals *i*∈ S, *i* = 1, 2, ..., n, as we observe $\left(y_i, v_i', w_i'\right)'$. We have assumed also the availability of sample proportions and means for the variables in z_i' which can be used to calibrate the sample weights, Kamun et. al. (July 28, 2021, August 31, 2021).

The process of obtaining the sample weight, which is the reciprocal of the sample inclusion probability, is found in Kamun et. al. (July 28, 2021, August 31, 2021). Below is the weight:

$$w_i = \tilde{P}^{-1} = 1/g(x_i) = 1/\exp\left(\frac{\left(x_i - mean_{x_i}\right)/sd_{x_i}}{\sqrt{\pi_i}}\right)$$

(4.1)

4.1Weighted Conditional Non Parametric estimator

Let $f_p\left(w_i | y_i, x_i, \theta\right)$ which is the conditional pdf for the weight in the population, be the weight of the population distribution where the observed weight stems from the sample distribution, f_s obtained by Bayes rule as

$$f_s\left(w_i | y_i, v_i, \theta\right) = f_p\left(w_i | z_i, \theta, i \in S\right) = \frac{\Pr\left(i \in S | y_i, v_i\right) f_p\left(w_i | y_i, x_i, \theta\right)}{\Pr\left(i \in S | y_i\right)} \quad (4.2)$$

We assume from equation (4.1) and in this study that

$$f_p\left(w_i | y_i, v_i, \theta\right) = f_p\left(w_i | y_i, x_i, \theta\right)$$

for each value of *i*. This is an example of informative sampling that should be accounted for in the inferential process. Equation (4.1) can be written equivalently as

$$f_s\left(w_i | y_i, v_i, \theta, \gamma, \beta\right) = \frac{E_S\left(w_i | v_i, \theta, \gamma, \beta\right) f_p\left(w_i | y_i, x_i, \theta, \beta\right)}{E_S\left(w | y_i, v_i, \gamma\right)} \quad (4.3)$$

where $E_S\left(w_i|.\right)$ refers to the conditional expectation of the weights w_i with respect to their sample distribution. Also γ is an unknown vector of the unknown regression coefficients of the regression model on the design variables and the outcome for sample weights. Following

$$G_S\left(w_i,\gamma,\beta\right)=\sum_{i\in S}\frac{\delta E_S\left(w_i|y_i,v_i,\theta,\gamma,\beta\right)}{\delta\beta} \quad (4.4)$$

is the sample log-likelihood score equation in relation to β, Kamun et. al.(July 28, 2021, August 31, 2021).

The process of obtaining the modified sample weight is found in Kamun et. al. (July 28, 2021, August 31, 2021). Below is the modified weight:

$$w_a=\frac{\delta E_S\left(w_i|y_i,v_i,\theta,\gamma,\beta\right)}{\delta\beta} \quad (4.5)$$

4.2 Non-parametric Rescaled weight (NPRW(I))

The process of obtaining the modified sample weight is found in Kamun et. al. (July 28, 2021, August 31, 2021). Below is the modified weight:

$$\tilde{w}_i^m=\left(\frac{w_{reg1}}{E\left(w_{reg1}|x_i,y_i,z_i\right)}\right)\left(\frac{E\left(w_{reg2}|x_i\right)}{w_{reg2}}\right)\left(\frac{w_i}{E\left(w_i|x_i\right)}\right) \quad (4.6)$$

4.3 Non-parametric Rescaled weight (NPRW(II)& NPRW(III))

The process of obtaining the sample weight, which is the reciprocal of the sample inclusion probability, w_i is found in Kamun et. al. (July 28, 2021, August 31, 2021). The Non Parametric Rescaled Weight (NPRW(II)) is obtained following the process given here. The absolute difference between the regressed observed and the observed value of the dependent variable and the conditional expectation of the difference between regressed and observed value of the dependent variable y on the predictor variables x_1 ,x_1 and z, y and z, and y, x_1 and z is first obtained. Then the product of the reciprocal of the sample inclusion probability and ratios of the absolute difference and conditional expectations is obtained to give the weight. Seen below:

$$\tilde{w}_i^{4m}=w_i\left(\frac{y_{regEst}-y_i}{E\left(y_{regEst}-y_i|x_i\right)}\right)\left(\frac{E\left(y_{reg2Est}-y_i|x_i,z\right)}{y_{reg2Est}-y_i}\right)\left(\frac{y_{reg3Est}-y_i}{E\left(y_{reg3Est}-y_i|y_i,z\right)}\right)\left(\frac{E\left(y_{reg1Est}-y_i|y_i,x_i,z\right)}{y_{reg1Est}-y_i}\right) \quad (4.7)$$

Meanwhile, the Non Parametric Rescaled Weight (NPRW(III)) is obtained following the process given here. The absolute difference between the regressed observed and the observed value of w_i and the conditional expectation of the difference between regressed and observed value of w_i on the variables x_1 ,x_1 and z, y and z, and y, x_1 and z is first obtained. Then the product of the reciprocal of the sample inclusion probability and ratios of the absolute difference and conditional expectations is obtained to give the weight. Seen below

$$w_d=w_i\left(\frac{w_{regEst}-w_i}{\mathrm{E}\left(w_{regEst}-w_i|x_i\right)}\right)\left(\frac{\mathrm{E}\left(w_{reg2.Est}-w_i|x_i,z\right)}{w_{reg2.Est}-w_i}\right)\left(\frac{w_{reg3.Est}-w_i}{\mathrm{E}\left(w_{reg3.Est}-w_i|y,z\right)}\right)\left(\frac{\mathrm{E}\left(w_{reg1.Est}-w_i|y,x_i,z\right)}{w_{reg1.Est}-w_i}\right) \quad (4.8)$$

4.4 Non-parametric weight (NPW(I) & NPW(II))

The process of obtaining the sample weight, which is the reciprocal of the sample inclusion probability, w_i is found in Kamun et. al. (July 28, 2021, August 31, 2021). The Non Parametric Rescaled Weight (NPW(I)) is obtained following the process given here. The conditional expectation of the difference between regressed and observed value of the dependent variable y on the predictor variables x_1, x_2, x_3 and $x_4$and the sum of the conditional expectation of the difference of the regressed and the observed values on x_1, x_2, x_3 and x_4, is first obtained. Then the product of the reciprocal of the sample inclusion probability and ratio of the conditional expectation of the difference between regressed and observed value of the dependent variable y on the predictor variables x_1, x_2, x_3 and x_4 and the sum of the conditional expectation of the difference of the regressed and the observed values on x_1, x_2, x_3 and x_4 is obtained to give the weight. Seen below:

$$w_c=w_i\frac{\mathrm{E}\left(\left(y_{.reg.Est}-y\right)|x_{1i},x_{2i},x_{3i},x_{4i}\right)}{\sum_{i=1}^{n}\mathrm{E}\left(\left(y_{.reg.Est}-y\right)|x_{1i},x_{2i},x_{3i},x_{4i}\right)} \quad (4.9)$$

The Non Parametric Rescaled Weight (NPW(II)) is obtained following the process given here. The conditional expectation of the difference between regressed and observed value of the , reciprocal of the sample inclusion probability $w_i$dependent on y, x_2, x_3 and x_4 and the sum of the conditional expectation of the difference of the regressed and the observed values of the reciprocal of the sample inclusion probability on y, x_2, x_3 and x_4, is first obtained. Then the product of the reciprocal of the sample inclusion probabilityw_i and ratio of the conditional expectation of the difference between regressed and observed value of the reciprocal of the sample inclusion probability on y, x_2, x_3 and x_4 and the sum of the conditional expectation of the difference of the regressed and the observed values of the reciprocal of the sample inclusion probability on y, x_2, x_3 and x_4 is obtained to give the weight. Seen below:

$$w_e = w_i \frac{\left(w_{reg.Est} - w_i\right)}{\mathrm{E}\left(\left(w_{reg.Est} - w_i\right) \middle| y_i, x_{2i}, x_{3i}, x_{4i}\right)} \tag{4.9}$$

4.5 Weights used for re-weighting estimators

The table below gives weights used to re-weight estimators starting with sample inclusion probability, $\tilde{P}_1$.

Weights used for re-weighting estimators			
s/n	Estimator	Plan - Type	Weight
1.	WLL	$\tilde{P}^{-1}$	$w_i = \tilde{P}^{-1} = 1/g\left(x_i\right) = 1/\exp\left(\frac{\left(x_i - mean_{x_i}\right)/sd_{x_i}}{\sqrt{\pi_i}}\right)$
2.	WCNPE(I)	w_a	$WCNP(I) = w_a = \frac{\delta E_S\left(w_i \mid y_i, v_i, \theta, \gamma, \beta\right)}{\delta\beta_i}$
3.	WCNPE(II)	w_b	$WCNP(II) = w_b = \frac{1}{\frac{\delta E_S\left(w_i \mid y_i, v_i, \theta, \gamma, \beta\right)}{\delta\beta_i}}$
4.	NPRW(I)		$NPRW(1) = \tilde{w}_i^{3m} = \left(\frac{w_{reg1}}{E\left(w_{reg1} \mid y_i, x_i, z\right)}\right)\left(\frac{E\left(w_{reg2} \mid x_i\right)}{w_{reg2}}\right)\left(\frac{w_i}{E\left(w_i \mid x_i\right)}\right)$
5. NPRW(II) = $\tilde{w}_i^{4m}$			$\tilde{w}_i^{4m} = w_i\left(\frac{y_{regEst} - y_i}{E\left(y_{regEst} - y_i \mid x_i\right)}\right)\left(\frac{E\left(y_{reg2Est} - y_i \mid x_i, z\right)}{y_{reg2Est} - y_i}\right)\left(\frac{y_{reg3Est} - y_i}{E\left(y_{reg3Est} - y_i \mid y_i, z\right)}\right)\left(\frac{E\left(y_{reg1Est} - y_i \mid y_i, x_i, z\right)}{y_{reg1Est} - y_i}\right)$
6. NPW(I)=w_c			$w_c = w_i \frac{\mathrm{E}\left(\left(y_{.reg.Est} - y\right) \middle| x_{1i}, x_{2i}, x_{3i}, x_{4i}\right)}{\sum_{i=1}^{n}\mathrm{E}\left(\left(y_{.reg.Est} - y\right) \middle| x_{1i}, x_{2i}, x_{3i}, x_{4i}\right)}$
7. NPRW(III)= w_d			$w_d = w_i\left(\frac{w_{regEst} - w_i}{\mathrm{E}\left(w_{regEst} - w_i \mid x_i\right)}\right)\left(\frac{\mathrm{E}\left(w_{reg2.Est} - w_i \mid x_1, z\right)}{w_{reg2.Est} - w_i}\right)\left(\frac{w_{reg3.Est} - w_i}{\mathrm{E}\left(w_{reg3.Est} - w_i \mid y, z\right)}\right)\left(\frac{\mathrm{E}\left(w_{reg1.Est} - w_i \mid y, x_i, z\right)}{w_{reg1.Est} - w_i}\right)$
8.NPW(II)=w_e			$w_e = w_i \frac{\left(w_{reg.Est} - w_i\right)}{\mathrm{E}\left(\left(w_{reg.Est} - w_i\right) \middle| y_i, x_{2i}, x_{3i}, x_{4i}\right)}$

4.6 Weights used for matching estimators

Weights used for matching estimators			
s/n	Estimator	Plan-Type	Weighting equation
1.	WLL	$\hat{P}^{-1}$	$w_i=1/g(x_i)$
2.	WCNPE(I)	w_a	w_a=WCNP(I)
3.	NPRWE(I)	$\tilde{w}_i^{3m}$	$\tilde{w}_i^{3m}$ =NPRW(I)
4.	NPRWE(II)	$\tilde{w}_i^{4m}$	$\tilde{w}_i^{4m}$ =NPRW(II)
5.	NPWE(I)	w_c	w_c =NPW(I)
6.	NPRWE(III)	w_d	w_d = NPRW(III)
7.	NPWE(II)	w_e	w_e = NPWE(II)

5. Finite Sample Properties of Estimators

The first property deals with the mean location of the distribution of the estimator.

Biasedness - The bias of an estimator is defined as:

$$Bias\left(\hat{\theta}\right)=E\left(\hat{\theta}\right)-\theta \tag{5.1}$$

where $\hat{\theta}$ is an estimator of θ, an unknown population parameter. If E $(\hat{\theta}) = \theta$ then the estimator is unbiased. If E $(\hat{\theta}) \neq \theta$ then the estimator has either a positive or negative bias. That is, on average the estimator tends to over (or under) estimate the population parameter.

A second property deals with the variance of the distribution of the estimator. Efficiency is a property usually reserved for unbiased estimators.

Efficiency - Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be unbiased estimators of θ with equal sample sizes . Then, $\hat{\theta}_1$ is a more efficient estimator than $\hat{\theta}_2$ if

$$\operatorname{var}\left(\hat{\theta}_1\right)<\operatorname{var}\left(\hat{\theta}_2\right) \tag{5.2}$$

6. Conclusion

This study has used and modified the reciprocal of the sample inclusion probability as weights with the sole aim of obtaining Non Parametric Weighted Estimators which are more relatively efficiency and have smaller sample bias when compared with the Classical Estimators such as the Horvitz-Thompson Estimator. The study has come up with four new estimators, the Non Parametric Rescaled Weighted estimator I & II and the Non Parametric Weighted Estimator I & II.

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American Journal of Theoretical and Applied Statistics 2021; 10(4): 202-207 http://www.sciencepublishinggroup.com/j/ajtas doi: 10.11648/j.ajtas.20211004.14 ISSN: 2326-8999 (Print); ISSN: 2326-9006 (Online)