

Research on Numerical Simulation Method of Rock Dynamic Fracture Toughness Test

Yuan Zhou, Yiming Zhao*

State-Owned Wuhu Machinery Factory, Wuhu, PR China

Abstract: In this paper, the finite element analysis software LS-DYNA is used to simulate the impact compression experiment of cracked Brazilian disc using the split Hopkinson compression bar (SHPB) device. The variation of dynamic stress intensity factor (K factor) at the crack tip is studied. Through the analysis of an example, the K factor solution method in dynamic numerical simulation is improved on the basis of the static numerical extrapolation method. The conclusions are as follows: (1) Aiming at the numerical simulation calculation of stress intensity factor of rock dynamic crack, this paper proposes an improved method of K -factor calculation, that is, using the displacement of the element on the crack line to replace the node displacement for calculation can effectively solve the nonlinear relationship of node displacement and improve the accuracy of the calculation results. (2) For the Brazilian disk sample with a radius of 50mm, the radius is divided into 25 pieces during grid division, and the density ratio is adjusted to 1:5. When the crack surface element size is 0.5mm, the calculated linear correlation coefficient is closest to 1, reaching 0.9537. Therefore, this grid division method can improve the calculation accuracy and save calculation time. (3) By comparing the K factors obtained under three conditions of rectangular wave, trapezoidal wave and triangular wave, it is found that the stress intensity factor K at the crack tip is in direct proportion to the area S surrounded by the loading wave, so it can be preliminarily determined that the stress intensity factor at the crack tip is in direct proportion to the loading load under dynamic conditions. The loading rate of trapezoidal wave is between triangular wave and rectangular wave, which is a common waveform. Therefore, trapezoidal wave is recommended for numerical simulation.

Keywords: Fracture Toughness, Stress Intensity Factor, Brazilian Disc, Numerical Simulation

1. Introduction

Rock is a typical material containing defects, that is, a large number of cracks and microcracks. The further development and evolution of cracks and microcracks in the process of load bearing lead to fracture failure. In geotechnical engineering, the crack initiation and expansion are dynamic in many cases, such as the earthquake resistance of buildings and structures, rock burst in high stress areas of large hydropower stations

and underground mining sites. When analyzing problems related to crack initiation and propagation in rock mass, rock dynamic fracture toughness is an extremely important parameter. Rock fracture toughness is used to characterize the ability of rock materials to resist crack propagation, and is an important physical quantity to describe the failure mechanism of rock materials. At present, the research on fracture toughness of rock materials is mostly limited to static load cases [1,2]. In fact, rock failure often happens instantaneously, such as rock burst, landslide and other natural disasters in mining and tunnel excavation, so it is more important to study the dynamic fracture of rock [3].

More than 70 years ago, Carneiro FLLB, a Brazilian scholar, first proposed to test the tensile strength of concrete using diametrically opposed compression disc specimens [4], which was named Brazil Experiment. Later, famous international societies defined it as a standard method for testing the tensile strength of brittle materials such as concrete or rock [5,6], and a large number of domestic scholars [7-9] introduced the Brazil disc experiment into testing the static or dynamic fracture toughness of rock/concrete. The use of numerical simulation method can greatly improve the efficiency of engineering research and save time and money for project research. Therefore, this paper uses the nonlinear finite element dynamic analysis software LS-DYNA to conduct numerical simulation research on the impact compression test of Brazilian disc specimen with a central straight crack loaded by SHPB device, explore an accurate and efficient numerical simulation method.

2. Establish Finite Element Model

The SHPB finite element model established in this paper is shown in Figure 1. The materials defined are linear elastic materials. The density of incident rod, transmission rod and cushion block is 7800kg/m³, the elastic modulus is 210GPa, Poisson's ratio is 0.3, the sample density is 2500kg/m³, the elastic modulus is 30GPa, and Poisson's ratio is 0.2. In the linear elastic fracture problem, the crack tip should have singularity, so it is better to use the element with singularity at the crack tip. In the static loading process, the 20 node degenerate singular isoparametric element (i.e. degenerate singular isoparametric element method) is

used. However, in the dynamic research, there is no 20 node solid element to use, and only the solid164 8-node ordinary element can replace the singular element with singularity. After the analysis of numerical simulation examples, The 8-node solid164 solid element is available in this numerical simulation. In order to ensure the reliability of the finite element calculation results, the grid near the crack tip of the Brazilian disk

should be divided more closely, but the triangular element cannot appear.

The displacement time history curve of the element on the crack surface, the contact force at both ends of the specimen, and the stress strain time history curve of the element on the rod solved by LS-DYNA/Solver are stored for the next step of dynamic stress intensity factor calculation at the crack tip.

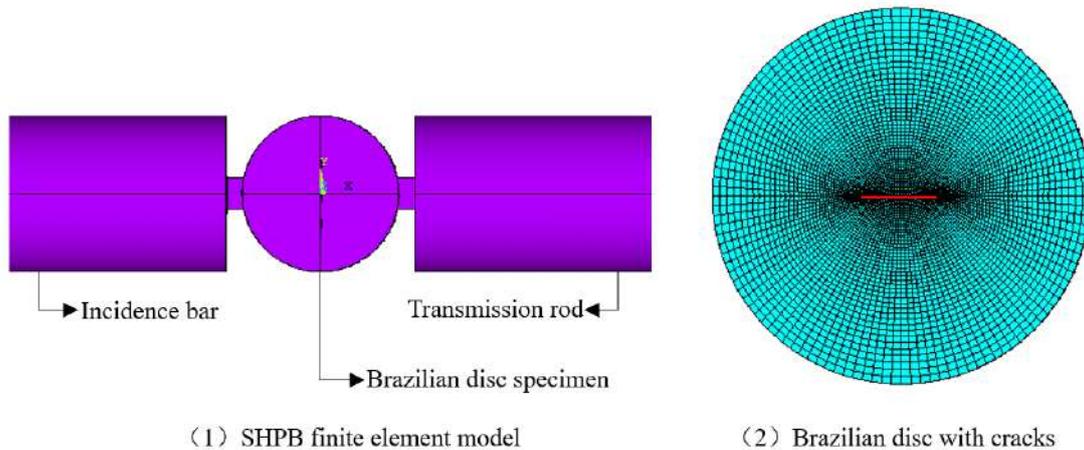


Fig. 1 SHPB finite element model and grid division of Brazilian disc specimen with cracks

3. Calculation of Stress Intensity Factor at Crack Tip

3.1. Calculation of K-factor Based on Numerical Extrapolation

Xutao Wu [10] proposed a new numerical calculation method of K-factor. The calculation results of a series of examples show that this method has high calculation accuracy and good stability. For plane stress crack problems, the error is only about 0.5%, and for three-dimensional crack problems, the error is about 1~5%. The displacement field of the research problem is obtained through finite element calculation. Take the displacement of the node on the crack surface and use the following formula to calculate the K-factor of each node:

$$K_i^* = \frac{E}{2} \sqrt{\frac{\pi}{2r_i}} v_i \tag{1}$$

In the formula, E is the elastic modulus, r_i is the distance from node i to the crack tip, and v_i is the displacement component of node i relative to the crack tip. For the problem of load and structure symmetry, E is a single node value. When the load or structure is asymmetric, it is half of the difference between the upper and lower crack surfaces relative to the node displacement component. Is the apparent stress intensity factor of the interpolation point i on the crack surface. According to the experimental and relevant numerical results [11], it is found that within a

considerable size range from the crack tip, the first term of the asymptotic displacement expression has a good linear relationship with r_i . On this basis, the K-factor of the crack tip is obtained from these values using the linear regression of the least squares method:

$$K_{Crack\ tip} = \frac{1}{n} (\sum K_i^* - c \sum r_i) \tag{2}$$

$$c = \frac{n \sum K_i^* r_i - \sum K_i^* \sum r_i}{n \sum r_i^2 - (\sum r_i)^2} \tag{3}$$

Here, n is the number of interpolation points, taking 4-5 points.

The fitting accuracy depends on the selection of fitting interval. The interval where each data coincides with an equal straight line should be selected as the extrapolation interval. The fitting accuracy is determined by testing the linear correlation coefficient. The closer the linear correlation coefficient r is to ± 1 , the higher the interpolation accuracy is. The expression of the linear correlation coefficient r_{Kr} is:

$$r_{Kr} = \frac{\sum (r_i - \bar{r})(K_i^* - \bar{K}^*)}{\sqrt{\sum (r_i - \bar{r})^2 \sum (K_i^* - \bar{K}^*)^2}} \tag{4}$$

In the formula, \bar{r} and \bar{K}^* are the average value of the distance from the interpolation node to the crack tip and the average apparent K factor, respectively.

3.2. Improvement of Dynamic K-Factor Calculation Method Based on Numerical Simulation

Because LS-DYNA software lacks 20 node singular

isoparametric element, the ordinary 8-node solid164 element is used for simulation, but the element at the crack tip is required to be dense enough to achieve sufficient accuracy. Elements on the crack surface (as shown in Figure 2).

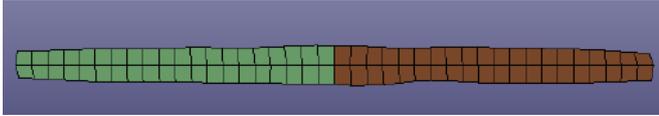


Fig. 2 Schematic diagram of Brazilian disc crack surface element

The crack tip nodes are subject to displacement constraints in the direction perpendicular to the crack surface (y-direction). Through some examples, the reason for such constraints is to maintain the symmetry of the displacement of the upper and lower nodes on the crack surface. At the same time, the crack tip nodes are constrained for the convenience of subsequent calculations, because the node displacements obtained later are relative to the crack tip displacements. Other nodes on the crack surface are not constrained, and the y-direction displacement d of 5 nodes A, B, C, D, E is taken from the left crack tip node (as shown in Figure 3).

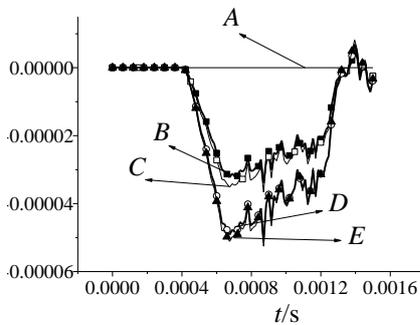


Fig. 3 Node Displacement Curve

It is obvious from Figure 3 that since the crack tip node (point A) is constrained, the displacement is always 0, and the displacement curves of two points B and C are similar. The same is true for two points D and E. If node displacement interpolation is used, 8 nodes are taken from the first node behind the crack tip on the crack surface, and their displacement time history curves are drawn from the LS-PREPOST post-processing program, saved, and then imported into the Origin software for data processing and substituted into the formula (2), (3) and (4). The crack tip stress intensity factor and correlation coefficient were obtained. As shown by the variation of apparent K-factor K_i^* with distance r_i (as shown in Figure 4), its linear correlation coefficient is poor and linear interpolation cannot be used.

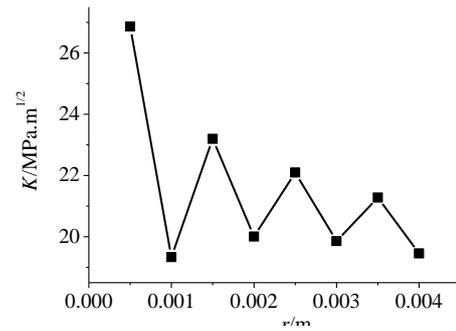


Fig. 4 Variation of apparent K-factor of nodes on crack surface with distance r

There are two nodes in an element on the crack surface (as shown in Figure 2), and the displacement time history curve of every two nodes is approximate, and the element displacement is the average of the displacement of the two nodes on the element. Therefore, it is proposed to replace the node displacement with the element displacement attached to the node on the crack line. The element displacement is the average of the displacement of the left and right nodes of the element, which is equivalent to the displacement of the midpoint of the element, that is, the displacement of the midpoint on the element is equal to the displacement of the element. In this way, the influence of the non linear relationship between the displacement time history curves of the two nodes in Figure 3, which are close to each other but differ a lot from the displacement time history curves of the other two nodes, can be offset.

Therefore, the definition of r_i in equations (2), (3) and (4) is changed to the distance from the intermediate node of the i th element to the crack tip, and the definition of v_i is changed to the displacement component of the i th element relative to the crack tip. From the first unit behind the crack tip unit, 8 units are taken in turn. (The reason why the crack tip unit is not taken is to obtain a better linear relationship.) The displacement time history curve is drawn from the LS-PREPOST post-processing program and saved. Then, the data is imported into the Origin software for data processing and substituted into the formula (2), (3), (4) to obtain the crack tip stress intensity factor and correlation coefficient. Draw the change of apparent K factor with distance as shown in Figure 5 below.

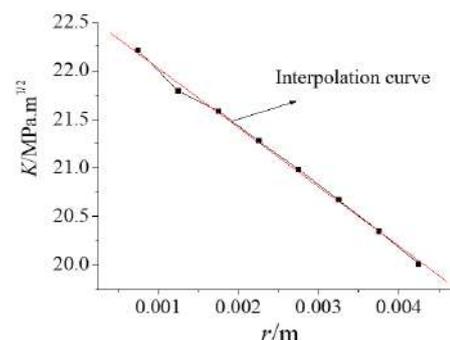


Fig. 5 Apparent K-factor of element midpoint on crack surface changes with distance r

It can be seen from Figure 5 that the linear properties of the results obtained in this way can meet the requirements and linear interpolation can be carried out. Therefore, it is feasible to replace the node displacement on the crack surface with the element displacement on the crack surface and calculate the stress intensity factor at the crack tip by imitating the formula (1), (2) and (3) of the degenerate singular isoparametric element method, and use formula (4) to determine the reliability of the results.

4. Suggestions on the selection of simulation process parameters

4.1. Gridding

As mentioned in the previous section, in order to ensure the reliability of the numerical simulation results, the unit division near the crack tip should be as dense as possible, but the unit grid division should not be too small, because they will greatly reduce the time step and avoid triangular elements as much as possible, because triangular elements are often rigid during deformation, Moreover, the mesh generation will have some influence on the dynamic finite element analysis. The division of samples is shown in Table 1 below.

Table 1 Grid Classification

Serial No	radius	pieces	density ratio	unit size	correlation coefficient r
1	50mm	25	1:10	1mm	0.9378
2	50mm	50	1:2	0.5mm	0.8811
3	50mm	25	1:5	0.5mm	0.9537
4	50mm	25	1:10	0.5mm	0.9263

Due to the influence of calculation time and physical memory, the grid is not divided more densely. The radius of 50mm is divided into 25 parts and 50 parts. Because the calculation time is not desirable, the grid of 50mm divided into 100 parts is not considered to be divided twice as densely. However, the cells near the crack tip are controlled to be denser than other places by controlling the density ratio of the cells, The stress intensity factor at the crack tip on the left side of the crack calculated for four meshing cases is shown in Figure 6. The following is a comparison of the four grid divisions, with the average value of the correlation coefficient as the comparison reference.

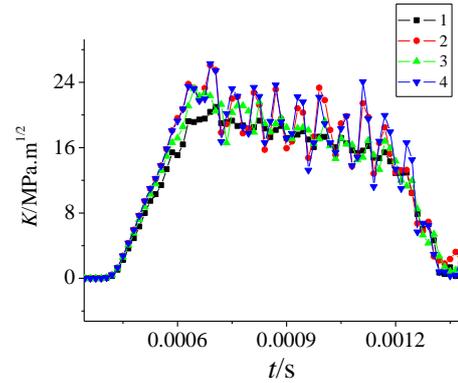


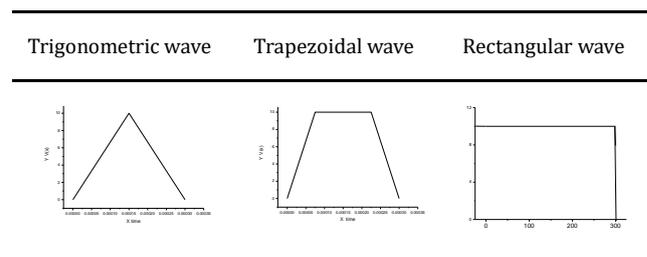
Fig. 6 Comparison of K-factor time history curves calculated under four grids

Based on the comprehensive analysis of Table 1 and Figure 6, the grid does have a certain impact in the dynamic simulation, but the impact is not too great. The grid division calculation result in the third case is the best. The denser the grid division is, the better the average value of the correlation coefficient is not necessarily. The more reliable the calculation result is. In order to control the impact of the grid as much as possible, the following numerical simulation analysis uses the third grid division scheme for grid division.

4.2. Selection of Loading Wave

For a long time, the effect of loading waveform on the mechanical properties of rock like materials has not been paid attention to. The effect of loading waveform on the mechanical properties of materials is essentially the difference of mechanical property response caused by different loading rates. Obviously, the load is added suddenly when the rectangular wave is loaded, which is equivalent to a pulse load, and the loading rate is large; The loading rate of triangular wave is a constant; The loading rate of each point is different when the sine wave is loaded. This paper studies the influence of three different loading waves on the dynamic stress intensity factor. Using the control variable method, three waveforms are applied to the same model. The three loading wave waveforms are shown in Table 2 below.

Table 2 Graph List of Three Loading Waves



The initial velocity of the loading wave load applied to the end node of the incident rod is 10m/s, and the time is 300us. The time history curve of the stress intensity factor at the crack tip under the action of three different waveforms is calculated, as shown in Figure 7 below.

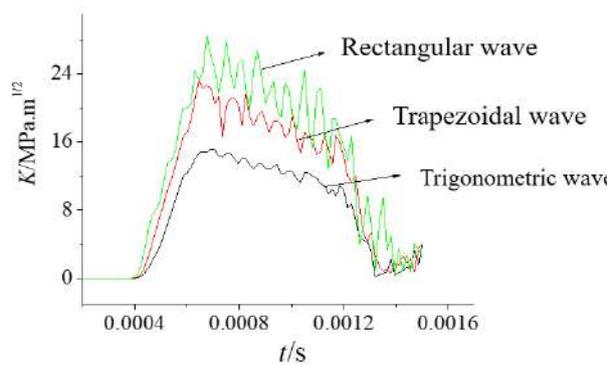


Fig. 7 Effect of Three Different Loading Waves on K-factor at Crack Tip

The calculation results are shown in Figure 7. When the loading wave is a triangular wave, the loading rate is a constant constant, the vibration amplitude of the time history curve of the stress intensity factor is the smallest, and the waveform oscillation is gentle. It can be seen that the amplitude of the time history curve of the stress intensity factor is greater when the loading wave is a rectangular wave than when the loading wave is a trapezoidal wave. It can be seen that the greater the loading rate is, the greater the stress intensity factor at the crack tip is.

In addition, the action effect of the loading wave is equivalent to the area S enclosed by the loading wave and the time axis. Therefore, the obtained crack tip K-factor should have a certain relationship with the area enclosed by the stress wave. Now take the data at a certain time point calculated by three kinds of loading waves for interpolation fitting, and derive the relationship between the crack tip stress intensity factor and the area enclosed by the loading wave. Interpolate with the area enclosed by the loading wave as the X axis and the crack tip K-factor at a certain time point as the Y axis (Figure 8), The linear correlation coefficient is 0.9991. Obviously, from the interpolation results, it can be determined that the stress intensity factor K at the crack tip is in direct proportion to the area S surrounding the loading wave, so it can be preliminarily determined that the stress intensity factor at the crack tip is in direct proportion to the loading load. The loading rate of trapezoidal wave is between triangular wave and rectangular wave, which is a common waveform. Therefore, trapezoidal wave is recommended for numerical simulation.

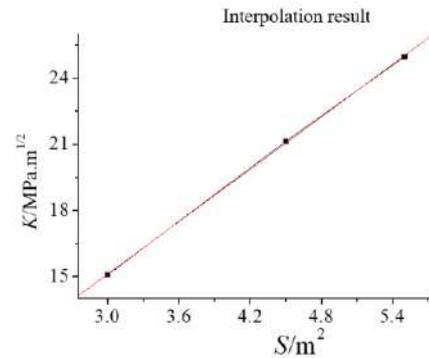


Fig. 8 Interpolation Results of K-factor and Loading Wave Area

5. Conclusion

- (1) Aiming at the problem of numerical simulation calculation of stress intensity factor of rock dynamic crack, this paper proposes an improved method of K-factor calculation, that is, using the displacement of the element on the crack line instead of the node displacement to calculate can effectively solve the problem of nonlinear relationship of node displacement and improve the accuracy of calculation results.
- (2) For the Brazilian disc sample with a radius of 50mm, the radius is divided into 25 pieces during grid division, and the density ratio is adjusted to 1:5. When the crack surface element size is 0.5mm, the calculated linear correlation coefficient is closest to 1, reaching 0.9537. Therefore, this grid division method can improve the calculation accuracy and save calculation time.
- (3) By comparing the K factors obtained under three conditions of rectangular wave/trapezoidal wave and triangular wave, it is found that the stress intensity factor K at the crack tip is in direct proportion to the area S surrounded by the loading wave, so it can be preliminarily determined that the stress intensity factor at the crack tip is in direct proportion to the loading load under dynamic conditions. The loading rate of trapezoidal wave is between triangular wave and rectangular wave, which is a common waveform. Therefore, trapezoidal wave is recommended for numerical simulation.

References

- [1] Jia Z, Castro-Montero A, Shah S P. Observation of Mixed Mode Fracture with Center Notched Disk Specimens[J]. Cement and Concrete Research, 1996, 26(1): 125-137. doi: 10.1016/0008-8846(95)00194-8
- [2] Dong S. Theoretical analysis of the effects of relative crack length and loading angle on the experimental results for cracked Brazilian disk

- testing[J]. *Engineering Fracture Mechanics*, 2008, 75(8):2575-2581.doi:10.1016/j.engfracmech.2007.09.008
- [3] Wei L I , Xie H P , Wang Q Z . Experimental Study for Dynamic Tensile Strength and Elastic Modulus of Marble using SHPB[J]. *Journal of Experimental Mechanics*, 2005, 20(2):200-206.doi:10.1080/02726340590910084
- [4] Carnero FLLB.A new method to determine the tensile strength of concrete//Proceedings of the 5th Meeting of the Brazilian Association for Technical Rules,3rd.Section,16 September 1943(in Portuguese),Rio de Janeiro,Brazil.126-129.
- [5] ASTM Standard. Standard test method for splitting tensile strength of intact rock core specimens. ASTM D3967-08.ASTM International, West Conshohocken, USA,2008.
- [6] ISRM Testing Commission. Suggested method for determining tensile strength of rock materials. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 1978, 15(3):99-103.
- [7] Wang Q Z , Li L , Wu L Z , et al. Improved Brazilian Test Specimen: From Flattened Brazilian Disc to Grooved Brazilian Disc. 2017.doi:10.6052/0459-1879-17-031
- [8] Wang Q Z , Li W , Xie H P . Dynamic split tensile test of Flattened Brazilian Disc of rock with SHPB setup[J]. *Mechanics of Materials*, 2009, 41(3):252-260.doi: 10.1016/j.mechmat.2008.10.004.
- [9] Ping Q , Ma Q Y , Yuan P . Energy dissipation analysis of stone specimens in SHPB tensile test[J]. *Caikuang yu Anquan Gongcheng Xuebao/Journal of Mining and Safety Engineering*, 2013, 30(3):401-407. doi: CNKI:SUN:KSYL.0.2013-03-017.
- [10] Tao W X , Yang B Y . Study on the numerical extrapolated method for calculating the 3 D crack front stress intensity factor[J]. *JOURNAL OF HEFEI UNIVERSITY OF TECHNOLOGY(NATURAL SCIENCE)*, 1999. doi: 10.1088/0256-307X/16/8/026.
- [11] Zhang S , Wang Q , Fan H . Study on modificatory coefficient in tensile strength formula for flattened Brazilian disk specimen[J]. *Dongnan Daxue Xuebao (Ziran Kexue Ban)/Journal of Southeast University (Natural Science Edition)*, 2005, 35:109-114.. doi:10.3321/j.issn:1001-0505.2005.z1.025.